Putting your data structure on a diet

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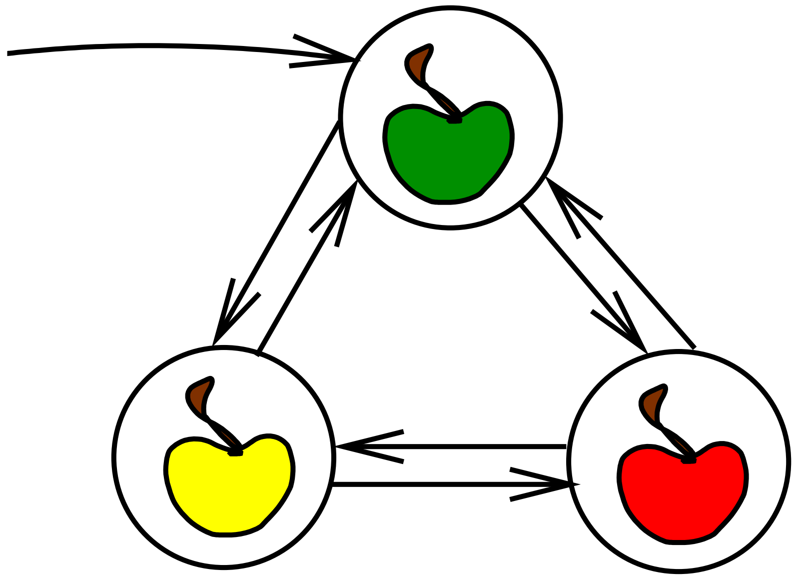
These slides are available at `http://www.cphstl.dk`

Memory overhead

- The amount of storage used by a data structure beyond what is actually required to store the elements manipulated (measured in words and/or in elements)
- We assume that pointers and integers occupy one word, and elements one or more words still being constant-sized objects

Example: Circular list of n apples; memory overhead $2n+O(1)$ words



n**:** # of elements currently stored

Research question

Q: How much can the memory overhead of a data structure be reduced without destroying its desirable properties?

A: Many data structures can be put on a diet so that, if the original memory overhead is $O(n)$, the memory overhead can be reduced to $O(n/\lg n)$, εn, or $(1+\varepsilon)n$ for any $\varepsilon > 0$ and sufficiently large $n > n(\varepsilon)$. The operations on the data structures are not slower, except by a small $O(1)$ factor or/and an additive term of $(1/\varepsilon)$.

True, for example, for

- lists (left as an exercise in the paper)
- ordered dictionaries (considered **today**)
- priority queues (presented in the paper)

Motivation

According to an earlier study [Brönnimann & Katajainen 2006], a red-black tree that has small memory overhead is faster than the implementation available at the C++ standard library for most operations. For further details, see [CPH STL Report 2006-1]

Performance ratio: Our programs were up to 1.2 times faster

Our ultimate goal is to develop library components that guarantee optimal time and space bounds

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Focus in this presentation

- Generality of the compaction technique
- Concrete examples

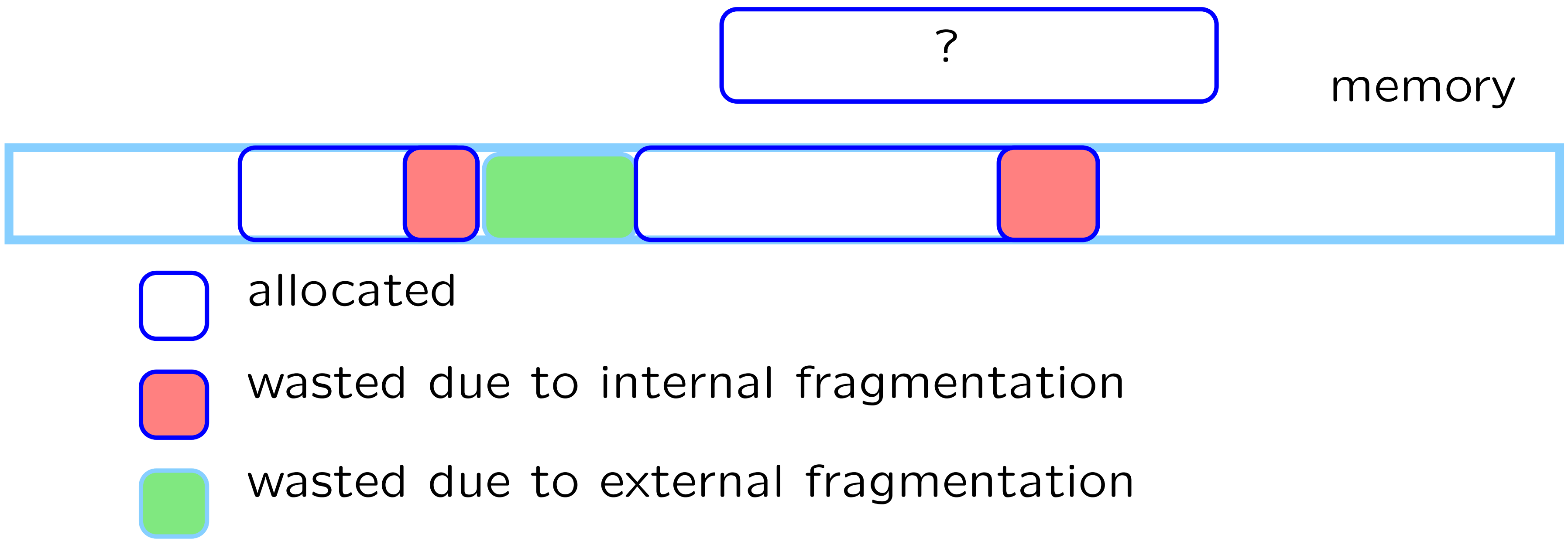
For technical details, see the forthcoming CPH STL report

Memory fragmentation

Allocation of memory segments of varying size can be problematic!

Internal fragmentation: Memory space allocated but not used

External fragmentation: Memory space that cannot be used because of disadvantageous allocation of memory segments



Minimum storage usage

Implicit data structures assume that there is an **infinite array** available to be used for storing elements; in practice, a **resizable array** should be used instead

Lower bound: A resizable array requires at least $\Omega(\sqrt{n})$ extra space for pointers and/or elements [Brodnik et al. 1999]

Upper bound: Realizations exist that require $O(\sqrt{n})$ extra space. Under a realistic model of dynamic memory allocation, the waste of memory due to internal fragmentation is $O(\sqrt{n})$ [Brodnik et al. 1999], even though external fragmentation can be large.

Earlier approaches

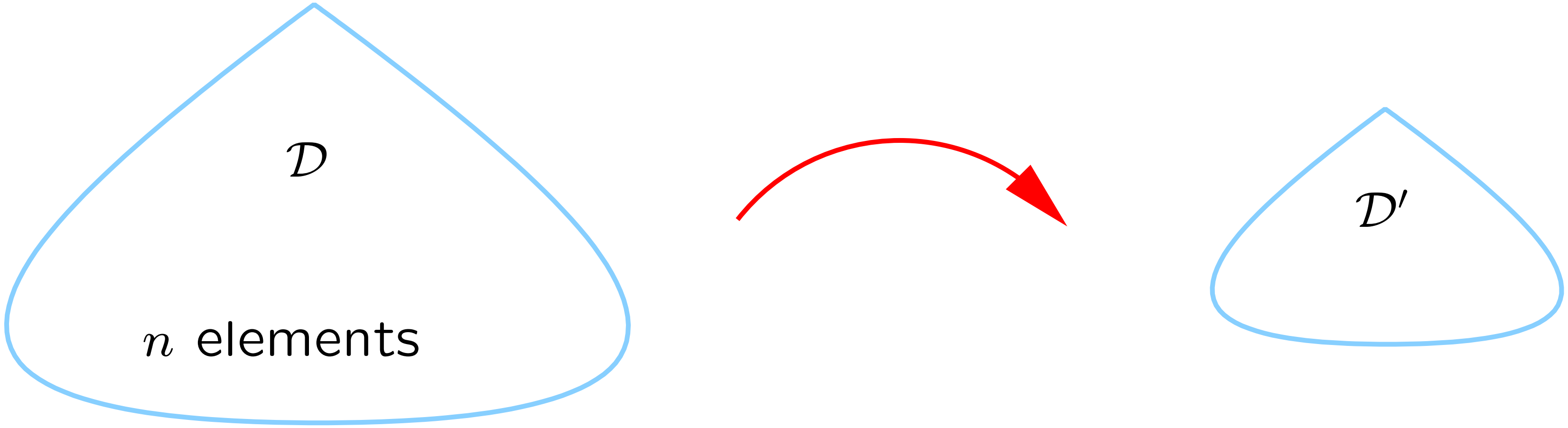
Ad-hoc designs: Improve the space efficiency of some specific data structures

Implicit data structures: Reduce the memory overhead to $O(1)$ words or $O(\lg n)$ bits

Often the developed data structures, like the searchable heap of Franceschini and Grossi [2003],

- are complicated,
- support a restricted set of operations, and
- do not provide certain desirable properties.

General data-structural transformation

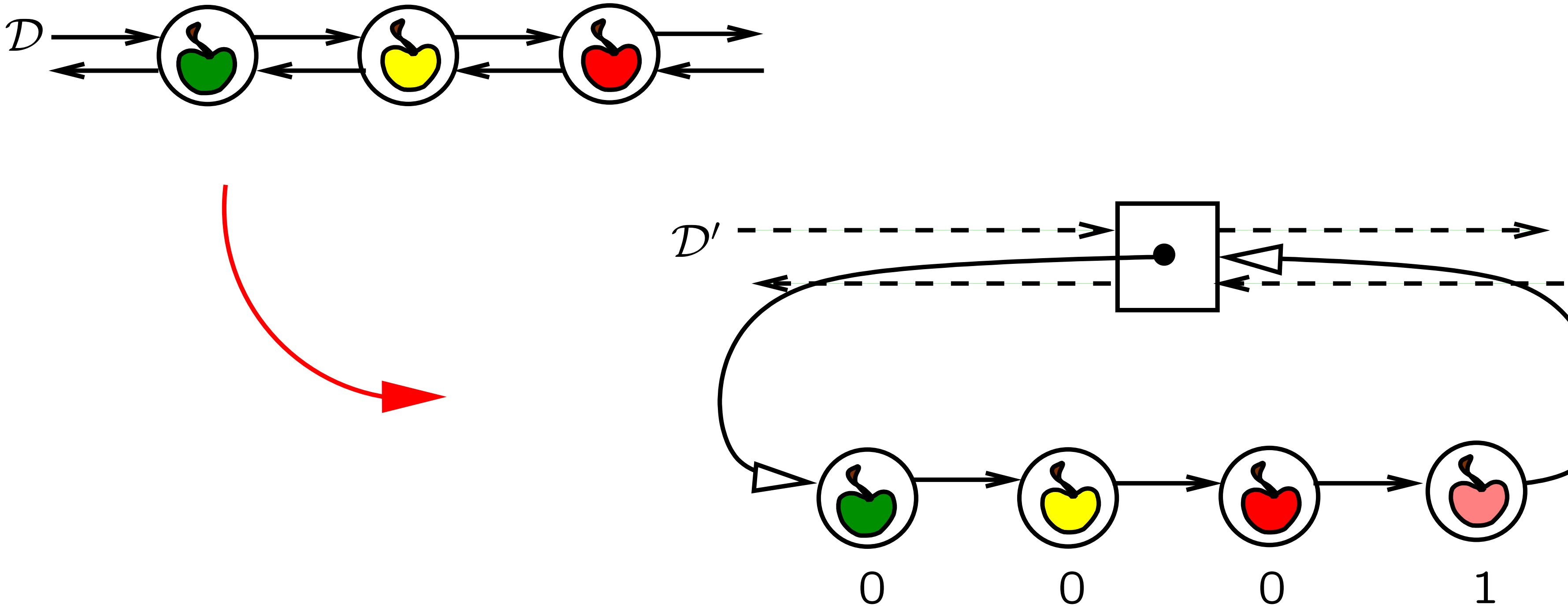


Memory overhead: $O(n)$ words

Memory overhead: $O(n/\lg n)$, εn, or $(1+\varepsilon)n$ words for any $\varepsilon > 0$ and $n > n(\varepsilon)$

Basic idea: Instead of operating on elements themselves, operate on groups—**chunks**—of $O(1/\varepsilon)$ elements

Doubly-linked lists



1 bit indicates the type of a node (last or not)

$b \mathinner{..} 4b$ elements per chunk, except one chunk

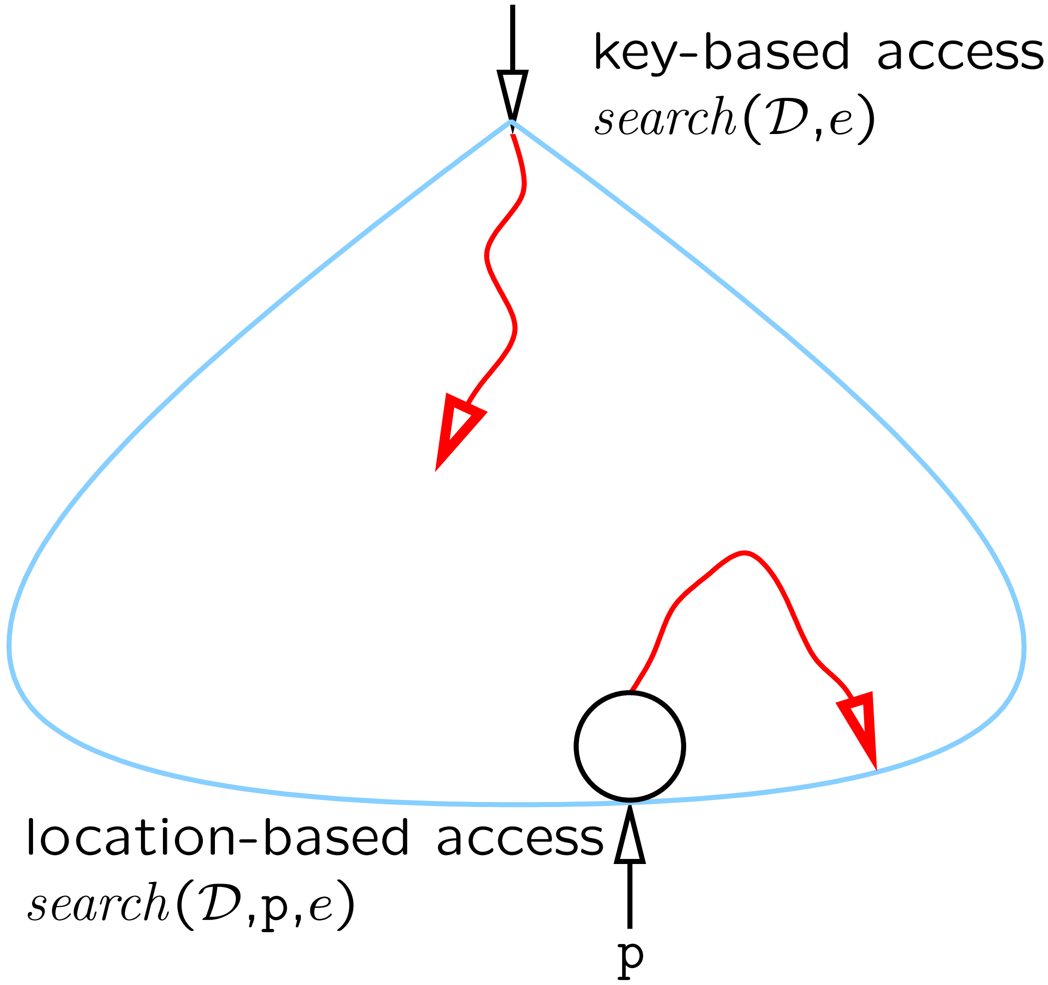
Memory overhead: $n + 3n/b + O(1)$ words, provided that bits can be packed in pointers

Bidirectional iterators: Iterator ++ is an additive term of $O(b)$ slower

Key-based/location-based access

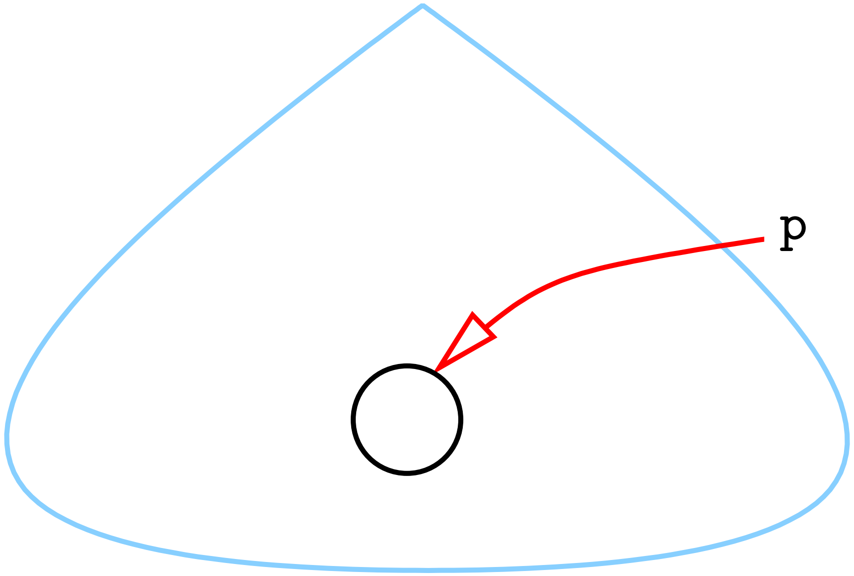
A data structure is called **elementary** if it only supports **key-based access**.

An important requirement often imposed by modern libraries is to provide **location-based access** to elements, as well as to provide **iterators** to step through a set of elements.



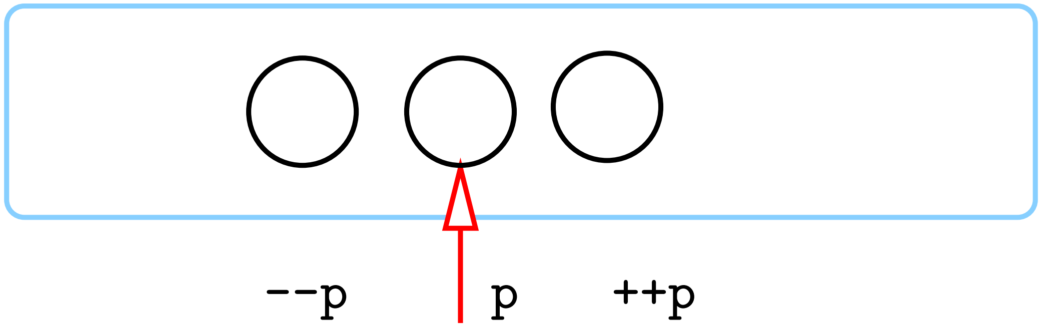
Locators and iterators

A **locator** is a mechanism for maintaining the association between an element and its location in a data structure.



Valid expressions: `X p; X p = q; X& r = p; *p = x; x = *p; p == q; p != q;`

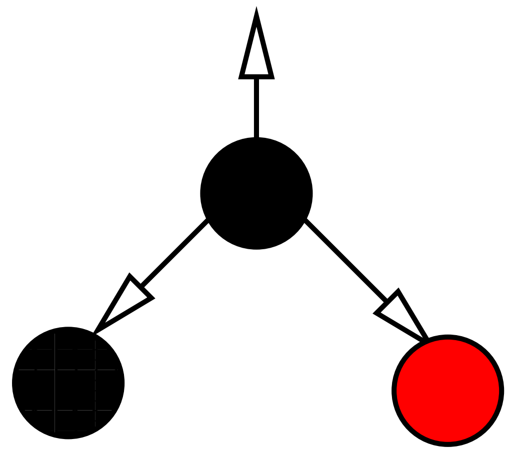
An **iterator** is a generalization of a locator that captures the concepts *location* and *iteration* in a container of elements



Bidirectional iterators: Locator expressions plus `++p` and `--p`

Red-black trees

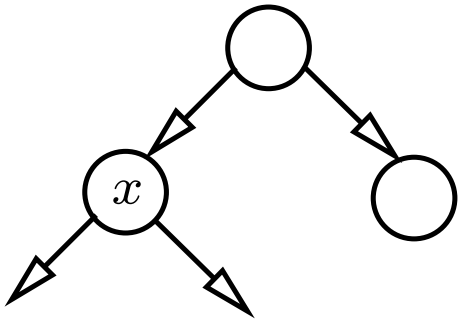
```
template <typename E>
struct node {
  node* child[2];
  node* parent;
  bool colour;
  E element;
};
```



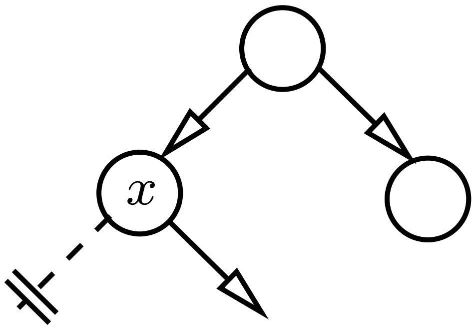
Memory overhead: $4n + O(1)$ words or more, because of word alignment

Immediate improvement: Pack the colour bits in pointers $\Rightarrow 3n + O(1)$ words [CPH STL Report 2006-1]

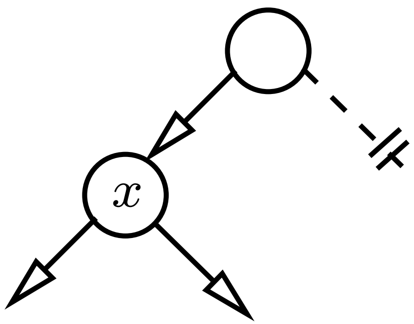
Child-sibling representation



x left child; sibling exists
x has left child
store left child & right sibling
access parent via sibling
access right child via left child

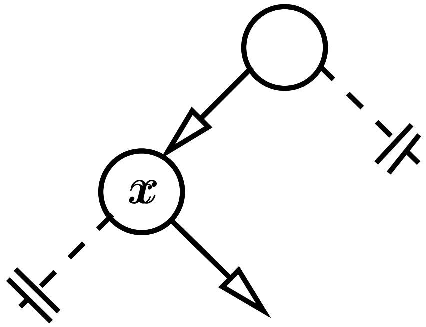


x left child; sibling exists
x has no left child
store right child & right sibling
access parent via sibling



x left child; no sibling exists
x has left child
store left child & parent
access right child via left child

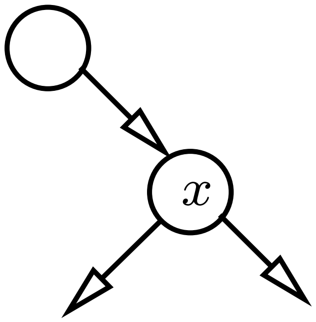
Child-sibling representation (cont.)



x left child; no sibling exists

x has no left child

store right child & parent

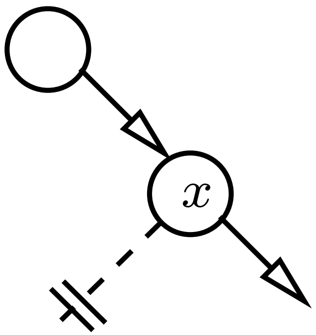


x right child

x has left child

store left child & parent

access right child via left child



x right child

x has no left child

store right child & parent

Child-sibling representation (cont.)

- 3 bits to indicate the type of a node
- 1 bit to indicate the colour of a node

Memory overhead: $2n + O(1)$ words, provided that the bits can be packed in pointers

Elementary dictionaries

Store the whole dictionary in an infinite array

$\mathcal{D}$:

- $S(n)$ and $U(n)$ time per search and update
- Memory overhead of $O(n)$ words
- All regularity requirements fulfilled

$\mathcal{D}'$:

- $S(n/\lg n) + O(\lg\lg n)$ and $O(S(n/\lg n) + U(n/\lg n) + \lg n)$ per search and update
- Exactly n locations for elements and at most $O(n/\lg n)$ locations for pointers and integers; furthermore, the whole dictionary can occupy a contiguous segment of memory

Nice theory: Freely movable data structures (e.g. circular array); D' works equally well for sets and multisets

Dictionaries with few iterators

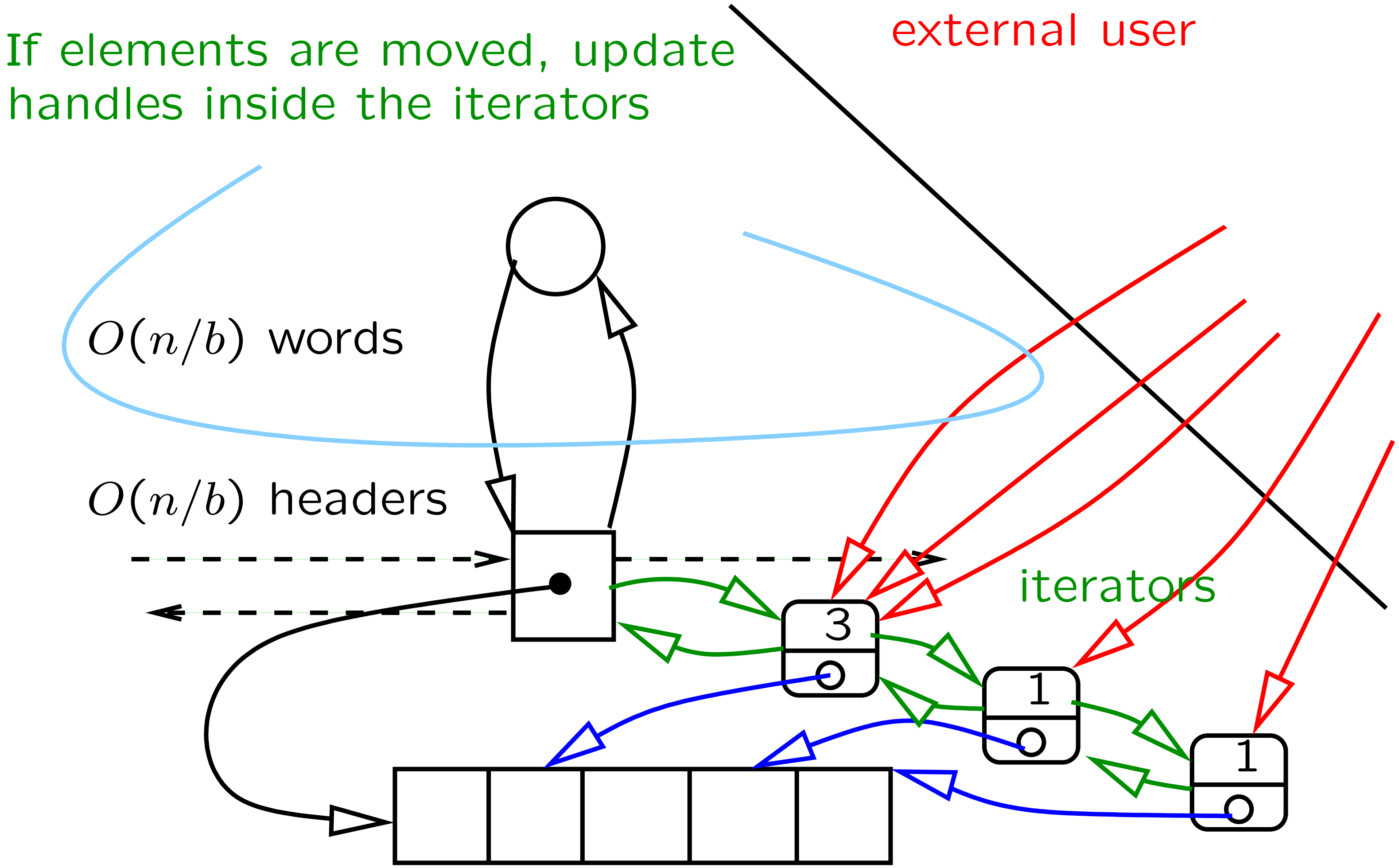
$\mathcal{D}$:

- $S(n)$ and $U(n)$ time per key-based/location-based search and update
- Memory overhead of $O(n)$ words
- Iterator operations in $O(1)$ time

$\mathcal{D}'$:

- $O(S(n/b) + \lg b)$ and $O(S(n/b) + U(n/b) + b)$ time per key-based/location-based search and update
- Memory overhead of $O(k + n/b)$ where k is the number of elements currently referenced by iterators
- Iterator operations in $O(1)$ time

Proof by picture



$b..4b$ elements per array; elements in sorted order

Dictionaries with many iterators

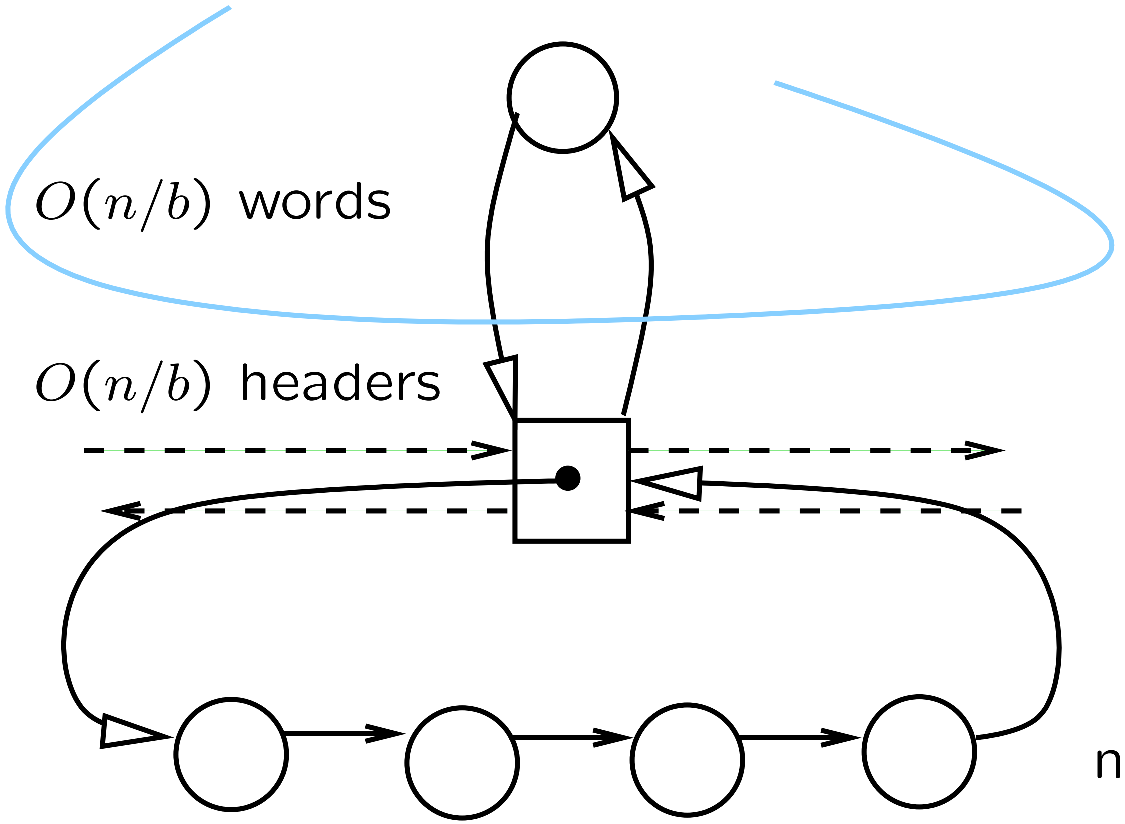
$\mathcal{D}$:

- $S(n)$ and $U(n)$ time per key-based/location-based search and update
- Memory overhead of $O(n)$ words
- Iterator operations in $O(1)$ time

$\mathcal{D}'$:

- $O(S(n/b) + \lg b)$ and $O(S(n/b) + U(n/b) + b)$ time per key-based/location-based search and update
- Memory overhead of n pointers plus $O(n/b)$ additional storage, provided that bits can be packed in pointers
- Iterator operations in $O(1)$ time, except that operator -- takes $O(b)$ time

Proof by picture



Iterators can be implemented as pointers to list nodes; packing of bits in pointers is again in use

Memory overhead:

$n + O(n/b)$ words

n list nodes

$b\,..\,4b$ elements per list; elements in sorted order

Regularity requirements

All transformations; requirements for $\mathcal{D}$:

Referential integrity: External references must be kept valid at all times

Location-based access: In case of multisets, location-based erase must be supported

Side-effect freeness: Let x, y, and z be three consecutive elements in $\mathcal{D}$. It should be possible to replace y with y', without informing $\mathcal{D}$, as long as $x \leq y' \leq z$.

Regularity requirements (cont.)

Transformations of elementary dictionaries; requirements for $\mathcal{D}$:

Little redundancy: Each element is stored only once at the place pointed to by its locator

Freely movable nodes: Every node knows who points to it so that, when it is moved, it can inform its neighbours

Constant in-degree: So that nodes can be moved in constant time

For example, a red-black tree fulfils all the requirements (if we disallow external references to nodes).

Conclusions

- Pointer packing may be a portability hazard
- It is disadvantageous to give raw pointers as locators for external users since such pointers make memory management difficult
- We would like to understand better data structures maintained in garbage-collected memory
- To be done: rigorous practical experiments