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Worst-case-efficient dynamic arrays in practice

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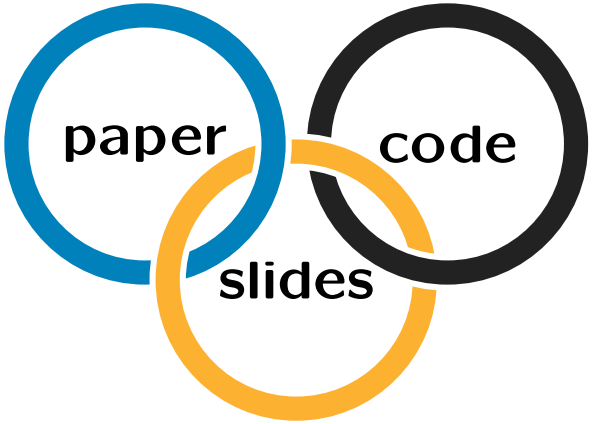

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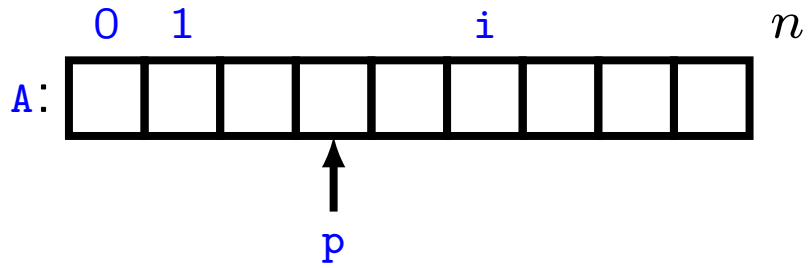
C array

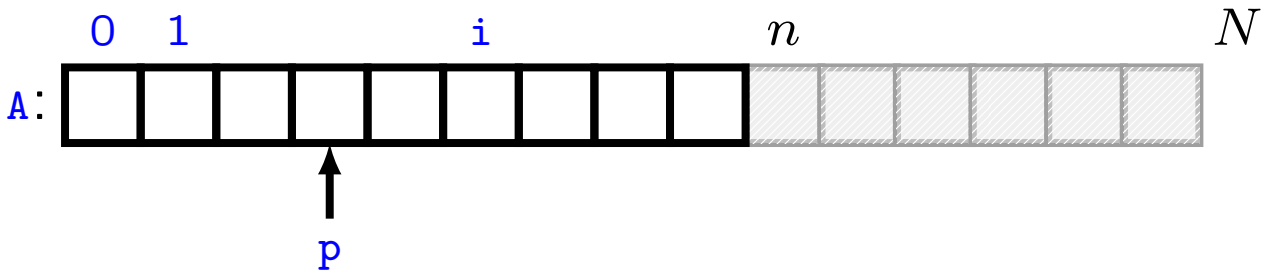


- contiguous memory segment
- size fixed (denoted n)
- uninitialized
- no bounds checking

- **fast random access**:
  `A[i]` $\equiv$ `*(A + i)`
- **iteration**:

```
p = A;
++p;
*p;
```

C++ array



- contiguous memory segment
- size varies (`++`,`--`, denoted n)
- capacity varies (denoted N)
- initialized (up to n)
- bounds checking if desired

- **fast random access**:

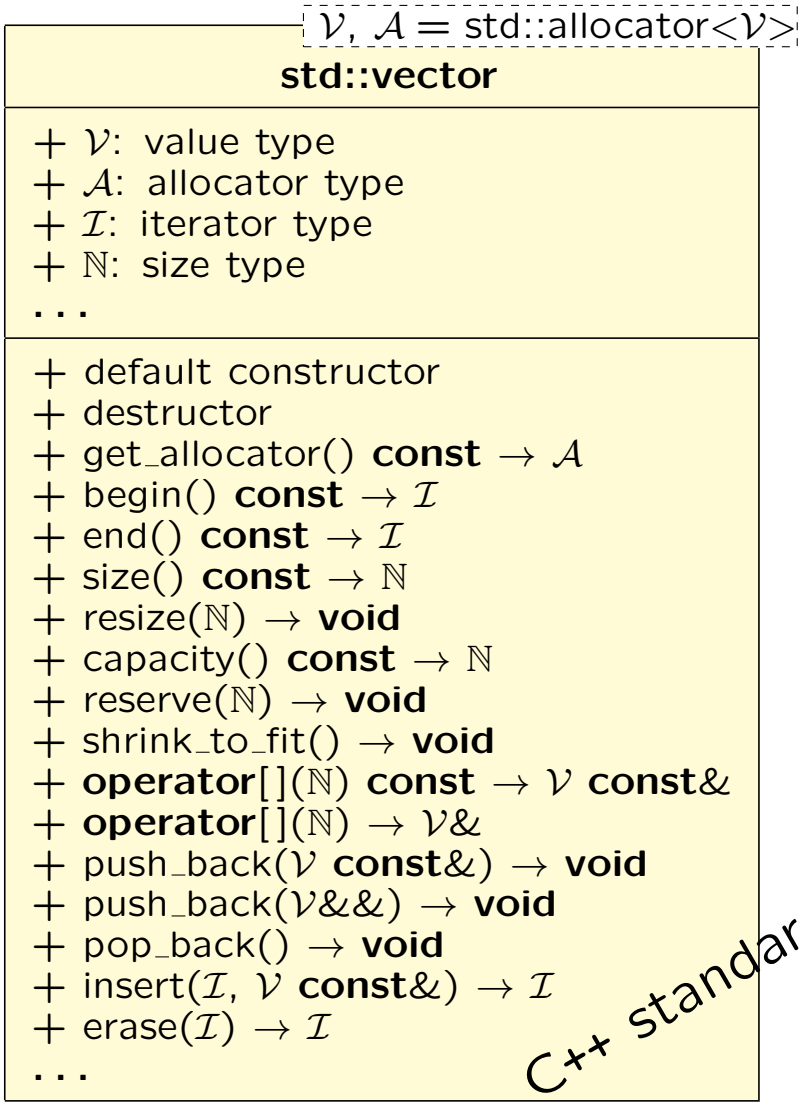
  `A[i]` $\equiv$ `*(A.begin() + i)`
- **iteration**:

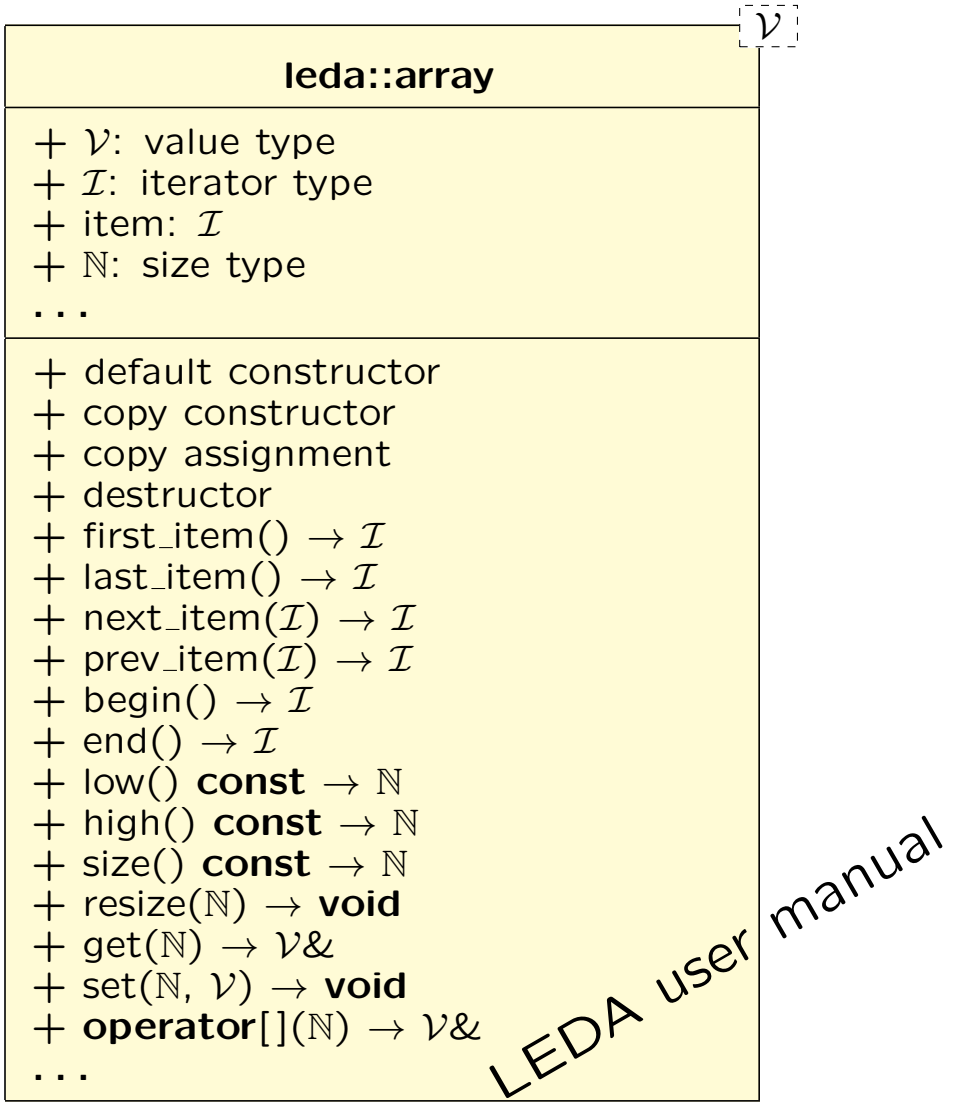
  `p = A.begin();`

  `++p;`

  `*p;`

Array interfaces



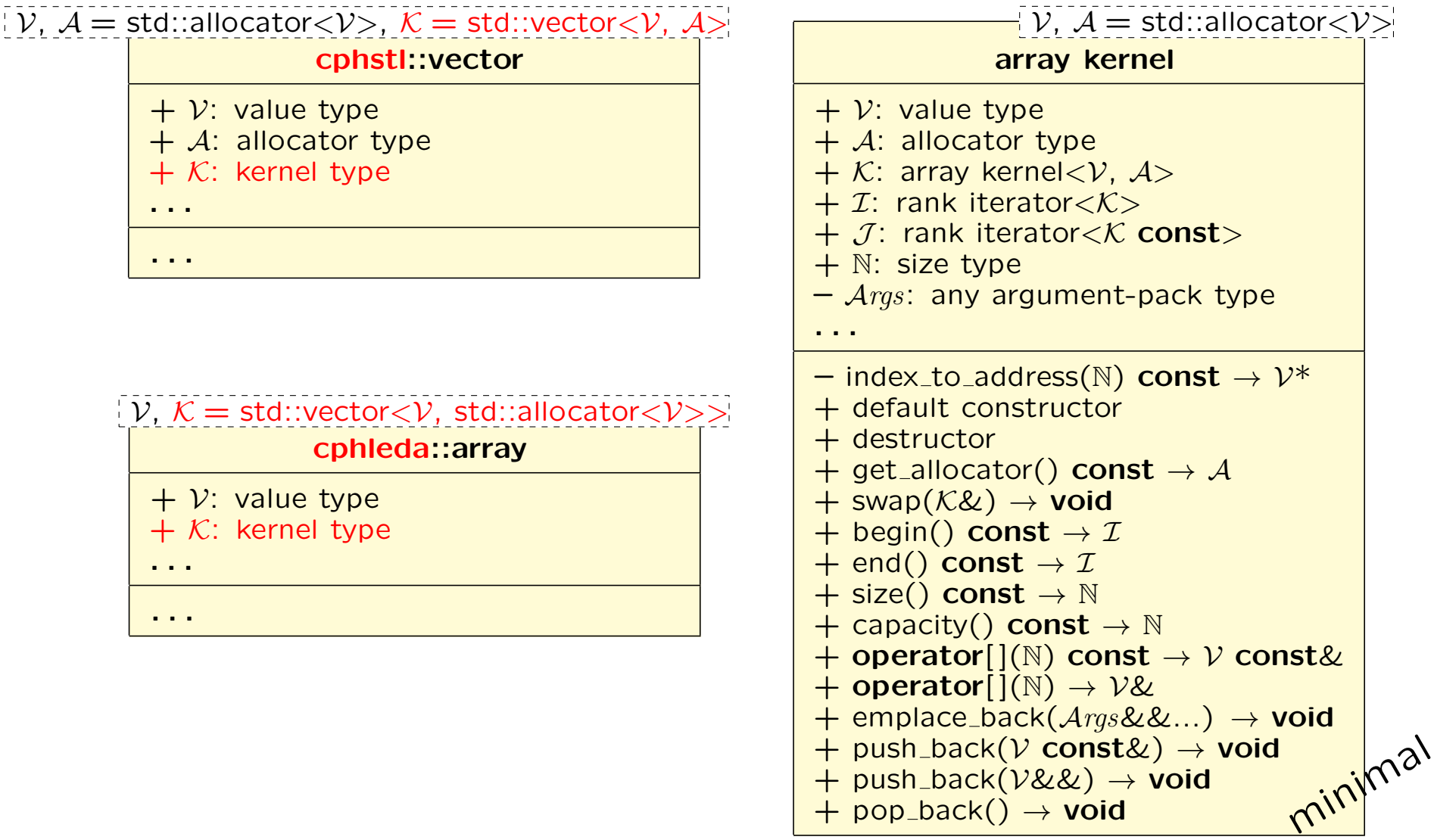


62 member functions

7 free functions

36 member functions

Bridge design pattern



Usage example

```
#include <algorithm> // std::sort
#include <cassert> // assert
#include <iostream> // standard streams
#include <memory> // std::allocator
#include "sliced_array.h++" // cphstl::sliced_array
#include "stl-vector.h++" // cphstl::vector

int main(int, char**) {
  using V = int;
  using A = std::allocator<V>;
  using S = cphstl::vector<V, A, cphstl::sliced_array<V, A>>;

  S sequence = {4, 2, 3, 1};
  std::sort(sequence.begin(), sequence.end());
  assert(std::is_sorted(sequence.begin(), sequence.end()));
  for (V x : sequence) {
    std::cout << x << " ";
  }
  std::cout << "\n";
  return 0;
}
```

```
jyrki@Janus:~/CPHSTL/Paper/Dynamic-arrays/Test$ make usage
g++ -std=c++11 -Wall -x c++ -I../Code usage.c++
./a.out
1 2 3 4
```

Reflection-based implementation

```
template <typename T>
class has_shrink_to_fit {

  template <typename U, void (U::*)()>
  struct check;

  template <typename U>
  static char member(check<U, &U::shrink_to_fit>*);

  template <typename U>
  static int member(...);

public:

  enum { value = sizeof(member<T>(nullptr)) == sizeof(char) };
};

template <typename V, typename A, typename K>
void vector<V, A, K>::shrink_to_fit() {
  shrink_to_fit_dispatch(typename std::conditional<has_shrink_to_fit<K>:: ↩
    value, std::true_type, std::false_type>::type());
}

template <typename V, typename A, typename K>
void vector<V, A, K>::shrink_to_fit_dispatch(std::true_type) {
  kernel.shrink_to_fit();
}

template <typename V, typename A, typename K>
void vector<V, A, K>::shrink_to_fit_dispatch(std::false_type) {
  // do nothing
}
```

Goal

In a minimal kernel, support all operations at $O(1)$ worst-case cost.

Assumptions: For allocators `a`, `b`, and integer $\Delta \geq 0$:

- `p = a.allocate(`Δ`)`: $O(1)$ worst case for any Δ
- `a.deallocate(p,`Δ`)`: $O(1)$ worst case for any Δ
- `a.construct(p,`$\bullet$`)`: $O(1)$ worst case
- `a.destroy(p)`: $O(1)$ worst case
- `a = b`: $O(1)$ worst case
- word-RAM primitives: $O(1)$ worst case

Memory-management tests

1. Repeat t times ($t = 10^6$):
   a) Allocate space for a block of Δ bytes ($\Delta = 2^k$).
   b) Deallocate the allocated block without using it.
2. Repeat the above r times ($r = 101$) and report the median of the execution times for a single allocate-deallocate pair.

memory-management tests; median of the execution times [ns]

`std::allocator<char> a; a.deallocate(a.allocate(Δ),Δ);`

2^5	2^6	2^7	2^8	2^9	2^{10}	2^{11}	2^{12}	2^{13}	2^{14}	2^{15}	2^{16}	2^{17}	2^{18}	2^{19}	2^{20}	2^{21}	2^{22}	2^{23}	2^{24}	2^{25}	2^{26}
35	35	61	61	60	67	68	66	66	68	68	68	50	51	52	52	52	51	51	52	2456	2644

`free(malloc(Δ));`

2^5	2^6	2^7	2^8	2^9	2^{10}	2^{11}	2^{12}	2^{13}	2^{14}	2^{15}	2^{16}	2^{17}	2^{18}	2^{19}	2^{20}	2^{21}	2^{22}	2^{23}	2^{24}	2^{25}	2^{26}
27	27	49	50	48	57	57	55	55	57	57	57	37	39	39	39	39	39	39	39	2360	2366

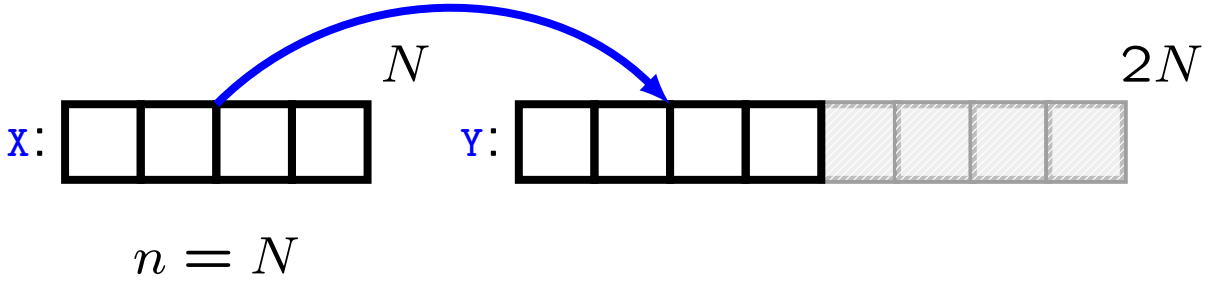
I know, I may have a problem, but you have it too!

Textbook solution

- **doubling**

if $(n = N)$

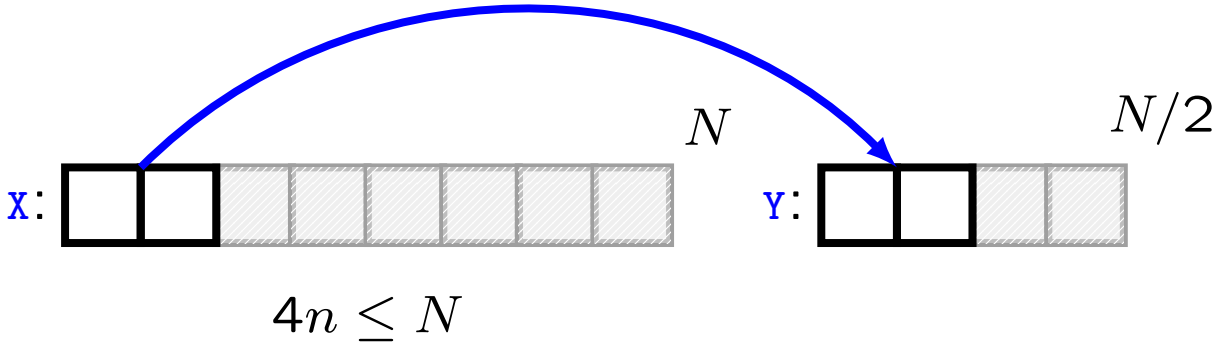
1. allocate $2N$
2. move all from `x` to `y`
3. release `x`



- **halving**

if $(4n \leq N)$

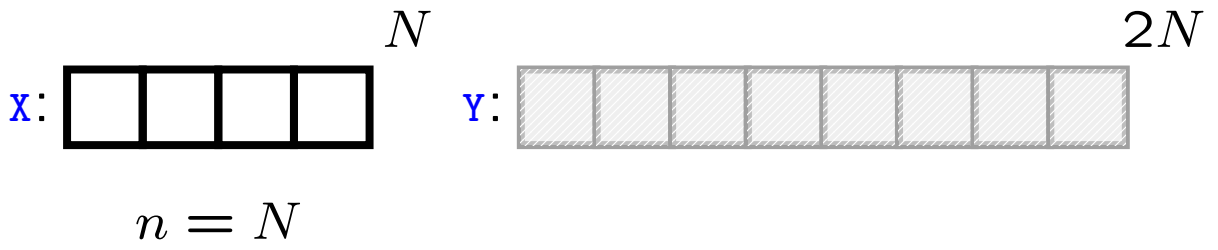
1. allocate $N/2$
2. move all from `x` to `y`
3. release `x`



Folklore solution (`cphstl :: resizable_array`)

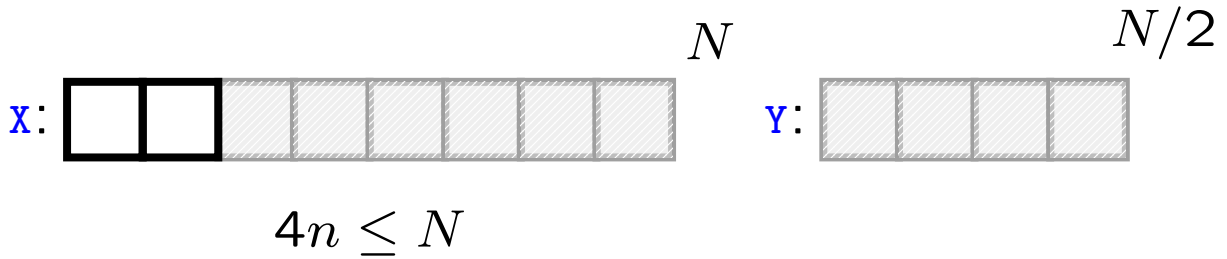
- **doubling**

if $(n = N)$
allocate $2N$



- **halving**

if $(4n \leq N)$
allocate $N/2$

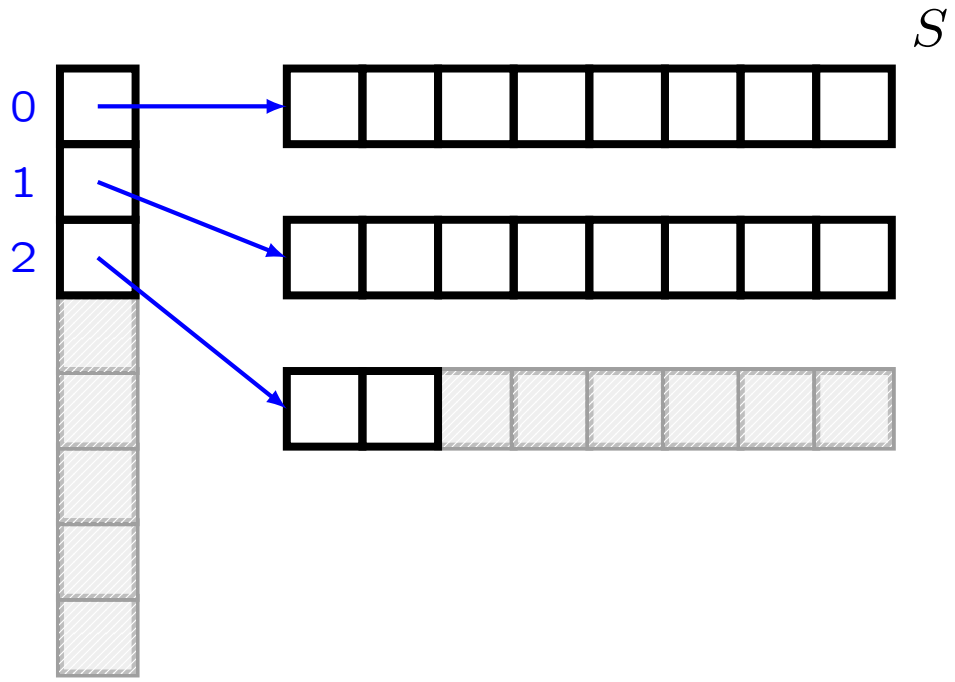


- **incremental copying**

`push_back`: move 1 from the end of `X` to `Y` at the same relative position
`pop_back`: move 2 from the end of `X` to `Y` at the same relative position
if (`X` empty)
release `X`

Slicing (`cphstl :: sliced_array`)

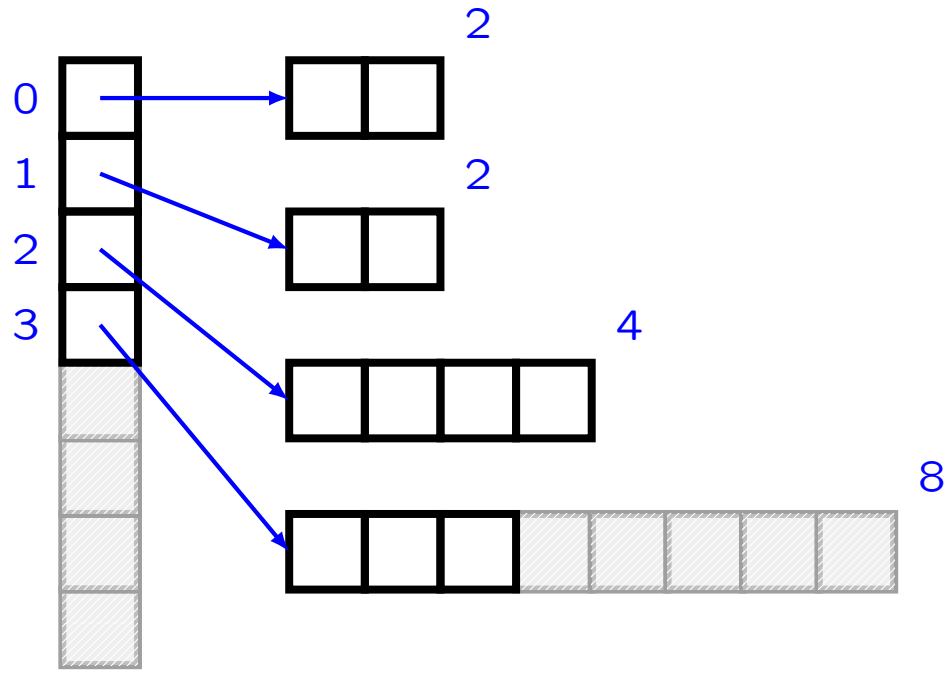
- S: slice size;
- **# slices**: $\lceil n/S \rceil$;
- **directory**: folklore;
- **last slice**: can be non-full



- **extra space**: at most S elements plus $O(n/S)$ pointers

Piling (`cphstl :: pile`)

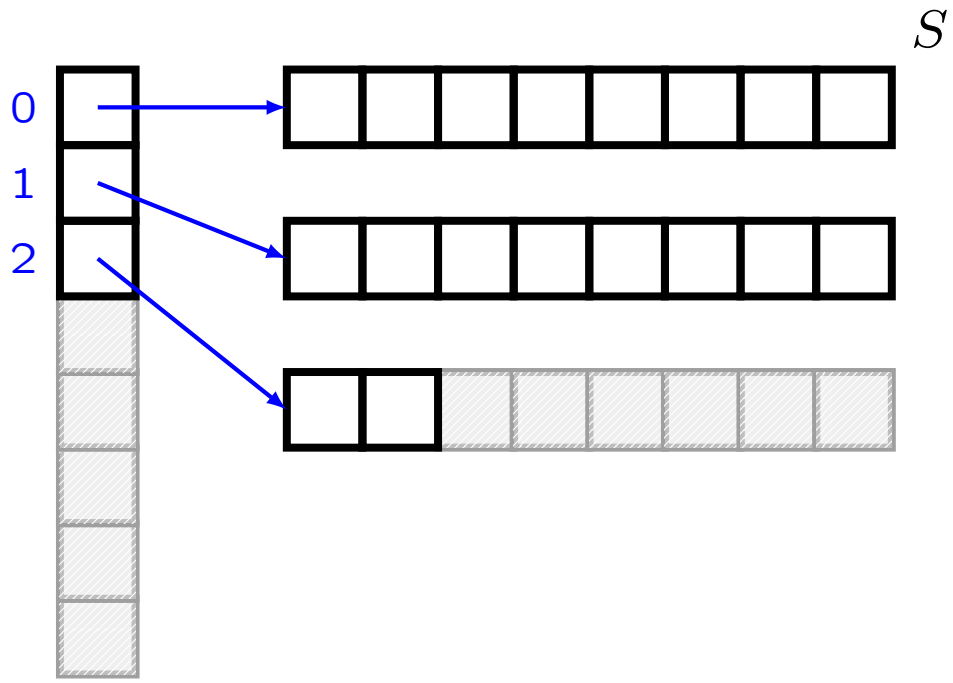
- slice sizes exponentially increasing; • **# slices**: $\lceil \lg(\max\{2, n\}) \rceil$;
- **directory**: folklore; • **last slice**: can be non-full



- **extra space**: at most n elements plus $O(\lg n)$ pointers

Hashed array tree

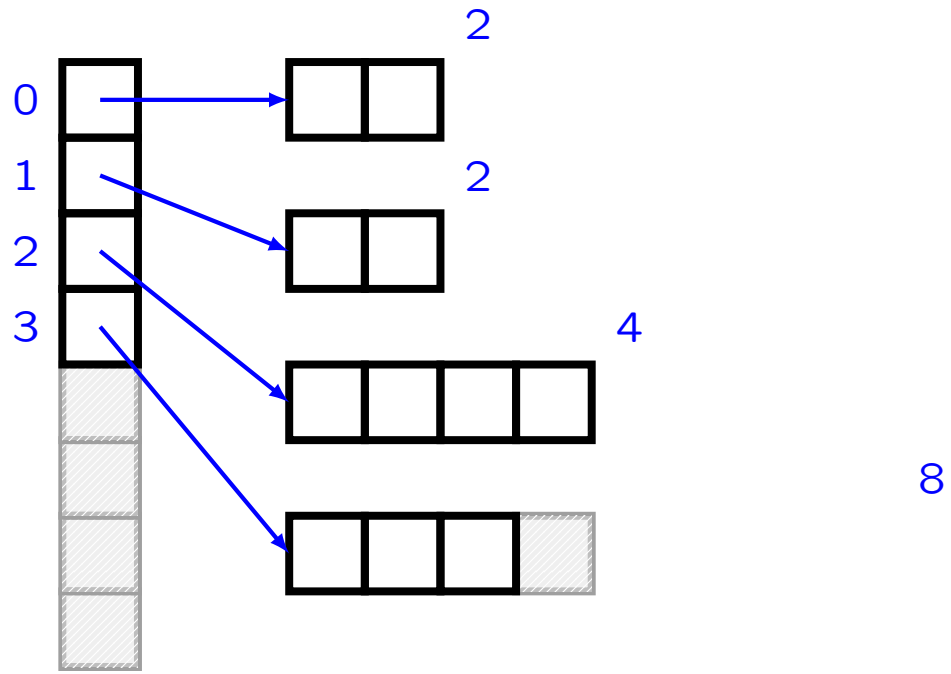
• N: fixed capacity; • $S = 2^{\lfloor \lg N/2 \rfloor} \in (\sqrt{N}/2 .. \sqrt{N}]$; • **# slices**: $\lceil n/S \rceil$; • **directory**: folklore; • **last slice**: can be non-full



• **extra space**: at most $\sqrt{N}$ elements plus $O(\sqrt{N})$ pointers

Piling hashed array trees (`cphstl :: space_efficient_array`)

• pile of hashed array trees; • **at level** i: hashed array tree of capacity $\max\{2, 2^i\}$; • **# levels**: $\lceil \lg(\max\{2, n\}) \rceil$; • **directory**: folklore; • **last slice** in the last hashed array tree: can be non-full



• **extra space**: at most $\sqrt{n}$ elements plus $O(\sqrt{n})$ pointers; space bound $\Omega(\sqrt{n})$ optimal

Summary of efficiencies

- S: slice size; • n: current size

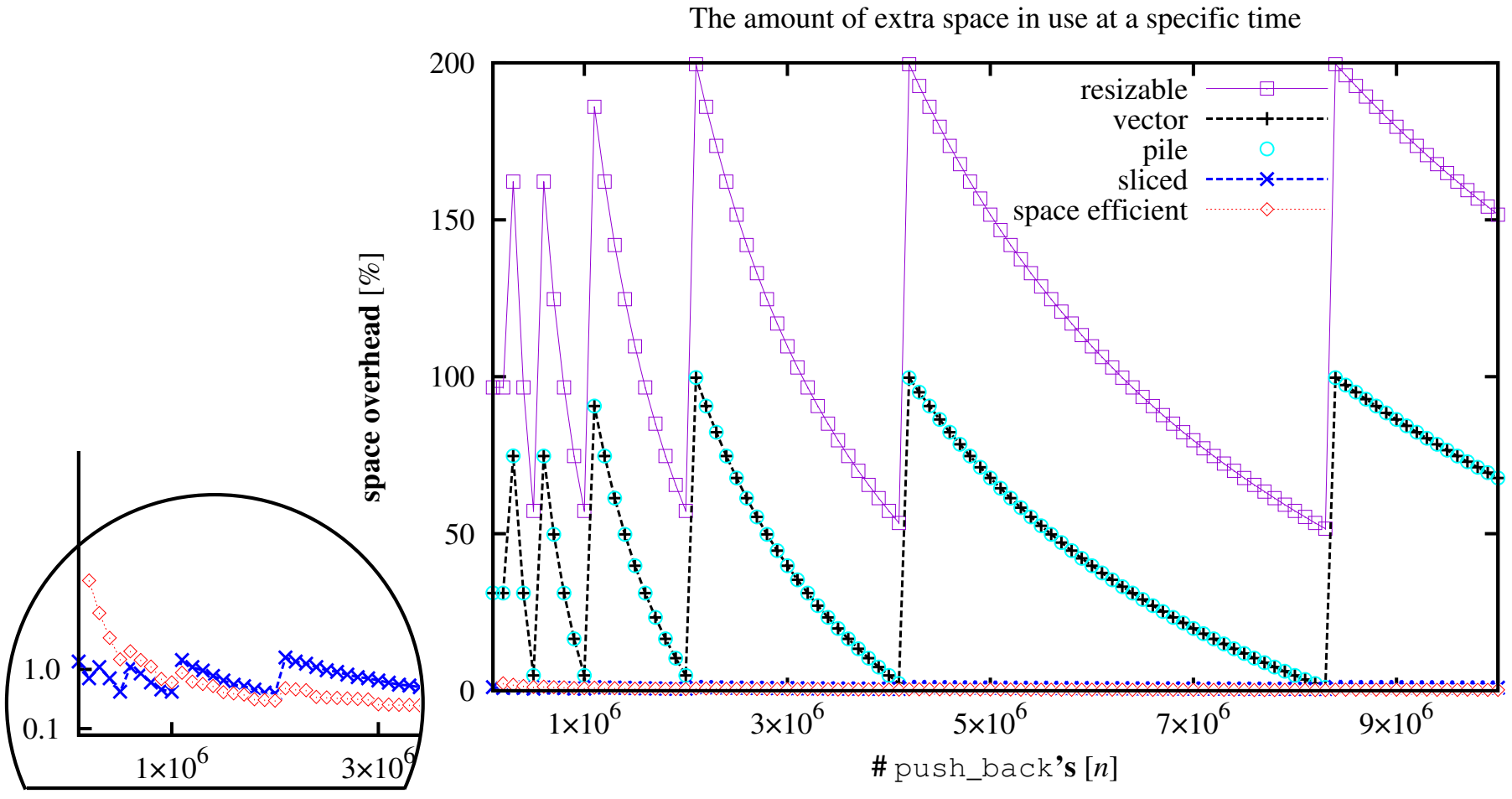
solution	operator [ ]	push_back | pop_back	elements	pointers
textbook	$O(1)$	$O(1)$ amortized	$6n$	$O(1)$
resizable	$O(1)$	$O(1)$	$12n$	$O(1)$
sliced	$O(1)$	$O(1)$	$n + S$	$O(n/S)$
pile	$O(1)$	$O(1)$	$2n$	$O(\lg n)$
space efficient	$O(1)$	$O(1)$	$n + \sqrt{n}$	$O(\sqrt{n})$

- **sources**: [Sitarski 1996]; [Brodnik, Carlsson, Demaine, Munro, Sedgewick 1999]; [Katajainen, Mortensen 2001]

Space test

- **overhead**: 100%·(space in use − n·**sizeof(int)**)/n·**sizeof(int)**

space overhead after n `push_back` operations



Subscripting operator

```
𝒱& operator[](ℕ i) {
  return *index_to_address(i);
}
```

contiguous array

```
𝒱* index_to_address(ℕ i) const {
  return A + i;
}
```

resizable array

```
𝒱* index_to_address(ℕ i) const {
  if (i < X_size) {
    return X + i;
  }
  return Y + i;
}
```

pile

```
ℕ whole_number_logarithm(ℕ x) {
  asm("bsr %0, %0\n"
      : "=r"(x)
      : "0" (x)
  );
  return x;
}
```

```
𝒱* index_to_address(ℕ i) const {
  if (i < 2) {
    return directory[0] + i;
  }
  ℕ h = whole_number_logarithm(i);
  return directory[h] + i - (1 << h);
}
```

sliced array

```
𝒱* index_to_address(ℕ i) const {
  return directory[i >> shift] + (i bitand mask);
}
```

space-efficient array

```
𝒱* index_to_address(ℕ i) const {
  if (i < 2) {
    return directory[0].index_to_address(i);
  }
  ℕ h = whole_number_logarithm(i);
  ℕ Δ = i - (1 << h);
  return directory[h].index_to_address(Δ);
}
```

Sorting tests

- **computer**: my Linux box; • **compiler**: `g++ -O3`; • **data**: `int`

introsort tests; running time per $n \lg n$ [ns]

n	vector	resizable	pile	sliced	space efficient
2^{10}	3.56	6.18	9.31	8.35	12.0
2^{15}	3.56	5.96	8.99	8.05	11.6
2^{20}	3.48	5.84	8.80	7.91	11.3
2^{25}	3.48	5.79	8.67	7.80	11.2

heapsort tests; running time per $n \lg n$ [ns]

n	vector	resizable	pile	sliced	space efficient
2^{10}	4.83	8.89	17.1	12.5	20.3
2^{15}	4.94	8.47	16.6	12.3	19.8
2^{20}	7.18	10.7	17.8	15.7	21.8
2^{25}	23.5	27.7	33.3	37.0	39.8

Modification tests

n `push_back` operations; running time per n [ns]

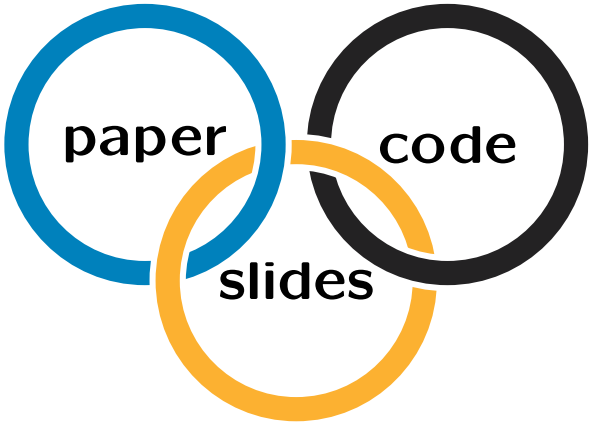
n	vector	resizable	pile	sliced	space efficient
2^{10}	4.23	5.18	5.65	4.65	10.3
2^{15}	3.52	6.39	5.16	4.63	7.35
2^{20}	4.78	8.48	5.12	4.60	6.92
2^{25}	4.15	8.42	4.55	4.58	6.75

n `pop_back` operations; running time per n [ns]

n	vector	resizable	pile	sliced	space efficient
2^{10}	0.0	3.62	3.08	2.56	8.15
2^{15}	0.0	2.99	2.15	2.60	5.55
2^{20}	0.0	2.86	2.27	2.41	5.17
2^{25}	0.0	2.91	2.11	2.43	5.07

Conclusions

- **worst-case efficiency**: an array cannot be a contiguous memory segment
- **insert**, **erase**: unnatural operations in this context
- **reflection-based implementation**: zero overhead
- **small kernels**: adaptability can be provided relatively cheaply.



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