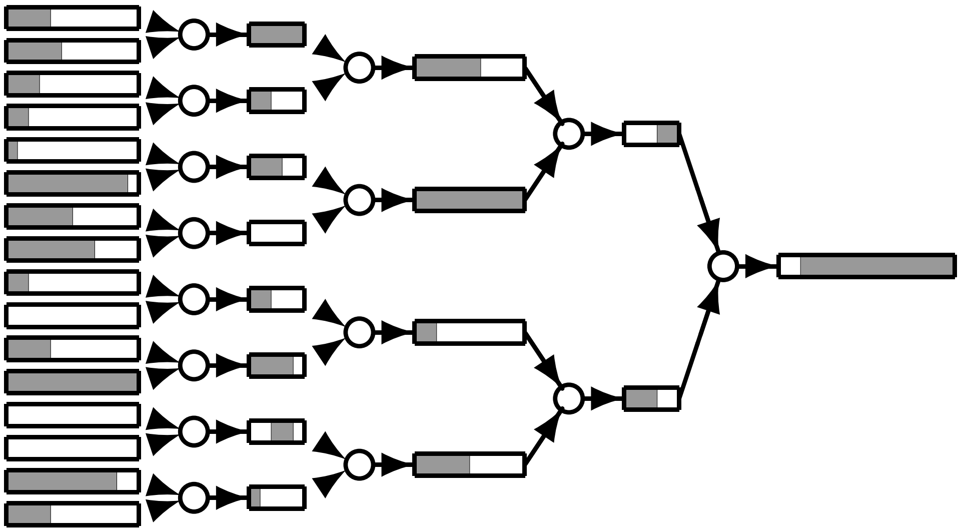
1 Funnel Heap

- Introduced by Brodal et al. [1].
- Based on merging instead of distribution.

References

[1] G. S. Brodal and R. Fagerberg, Funnel heap—a cache oblivious priority queue, *Proceedings of the 13th Annual International Symposium on Algorithms and Computation*, *Lecture Notes in Computer Science* **2380**, Springer-Verlag (2002), 426–438.

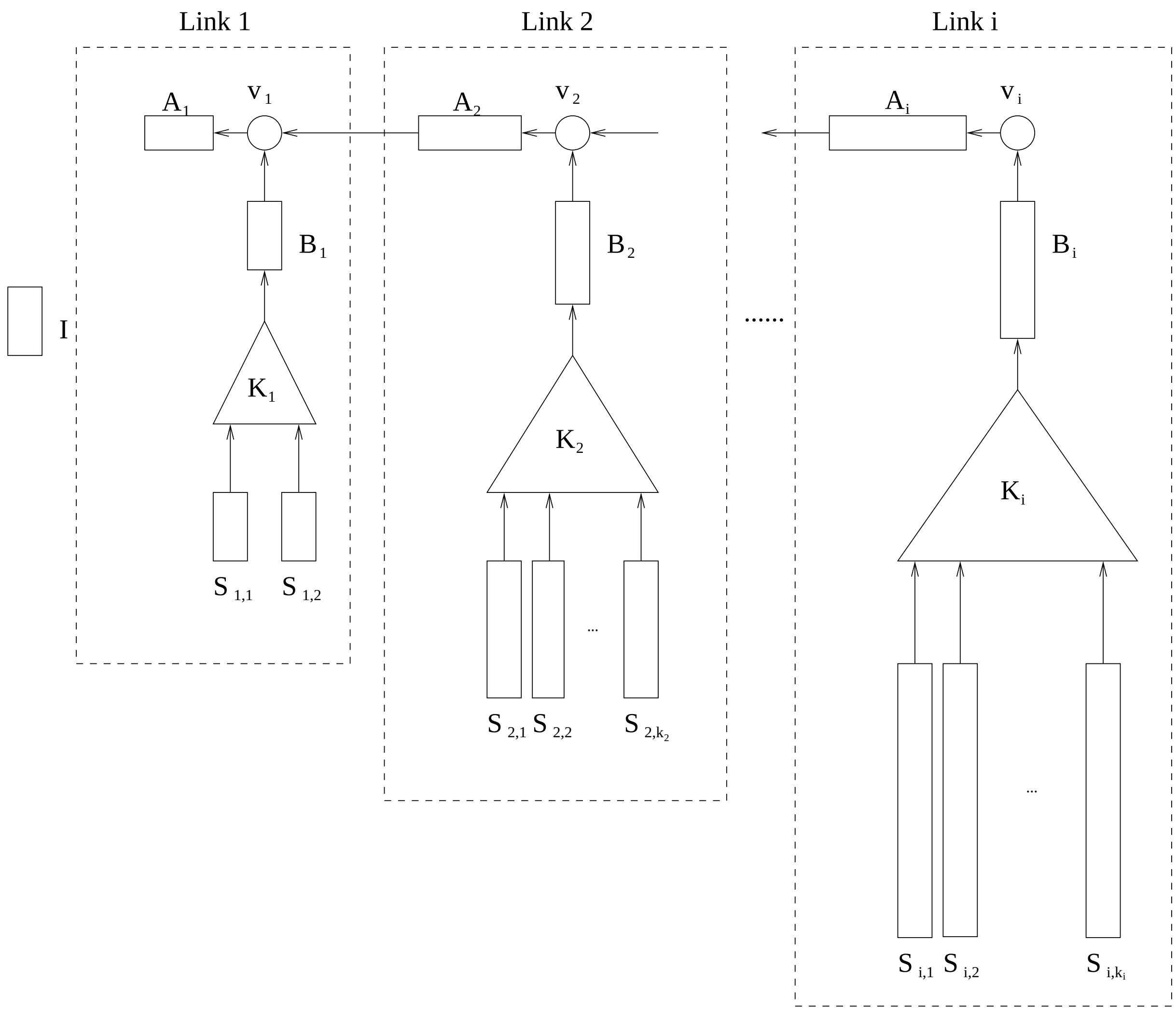
2 Funnel



3 Structure

$$
\begin{aligned}
(k_1, s_1) &= (2, 8), && (1)\\
s_i &= s_{i-1}(k_i + 1), \text{ and} && (2)\\
k_i &= \lceil\lceil {s_i}^{1/3} \rceil\rceil, && (3)
\end{aligned}
$$

where $\lceil\lceil x \rceil\rceil$ is "x rounded to the nearest power of 2".


Link 1
Link 2
Link i
A1
v1
A2
v2
Ai
vi
B1
B2
Bi
I
......
K1
K2
Ki
S 1,1
S 1,2
S 2,1
S 2,2
S 2,k2
S i,1
S i,2
S i,ki

4 Extract

- Fill A_1 if it is empty.
- Extract from A_1

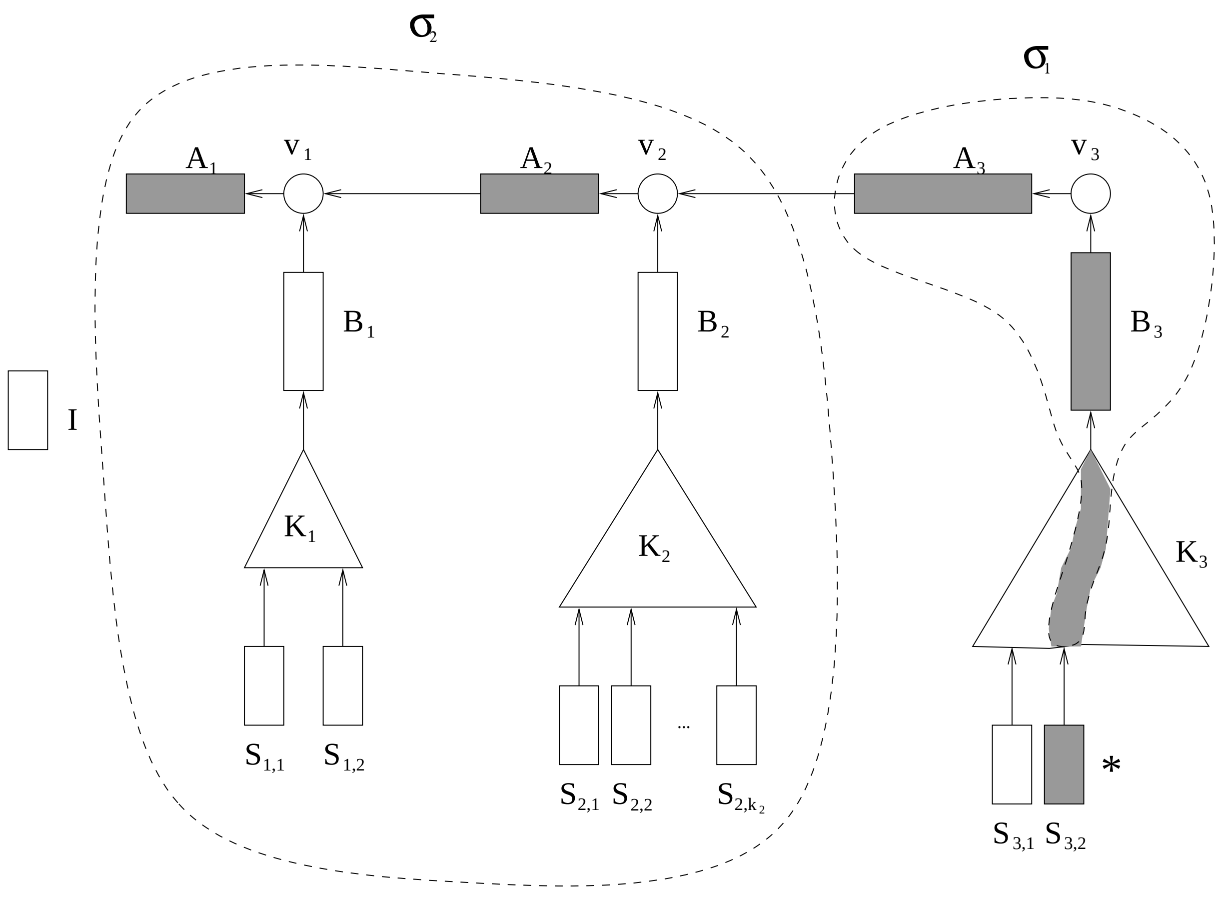
5 Insert

- Insert element in I.
- If I is full perform sweep.

6 Sweep

Idea: Merge elements, and move them to a higher link.

1. Find first empty S buffer.
2. Create σ_1 and σ_2, and merge them to form σ.
3. Insert elements on path from the root to the empty S buffer.



7 Space Complexity

- In a link i there are k_i S buffers of size s_i.
- By the definition of k and s we know that: $k_i = O(s_i^{1/3})$.
- The space consumption of the S buffers are thus $O(k_i^4) = O(s_i^{4/3})$.
- Since K_i, A_i and B_i are all of size $O(s_i)$, the space in the S buffers dominates the space used for a link i.
- Since s_i and k_i is increasing, the space consumption of a Funnel Heap with i links is dominated by the ith link.
- Since the space consumption of a Funnel Heap with i links is $O(k_i^4)$, and $k_{i+1} = O(k_i^{4/3})$, the worst case space complexity is $O(N^{4/3})$.

8 I/O Complexity

They prove that the amortized I/O complexity of an extract is

$$O\left(\frac{1}{B}\log_{M/B}\left(\frac{N}{B}\right)\right).$$

Which is optimal for cache-oblivious priority queues.