Helsinki, 8 December 2003

Title:
The current truth about heaps

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Co-workers:
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This talk is about the heaps we all love. I will explain how the heap functions are implemented in the CPH STL program library. The main contribution of the work done by my co-workers and myself is an experimental evaluation of various heap variants proposed in the computing literature. We have also done micro-benchmarking which gives some directions for future research.

These slides are available at
`http://www.cphstl.dk/`.

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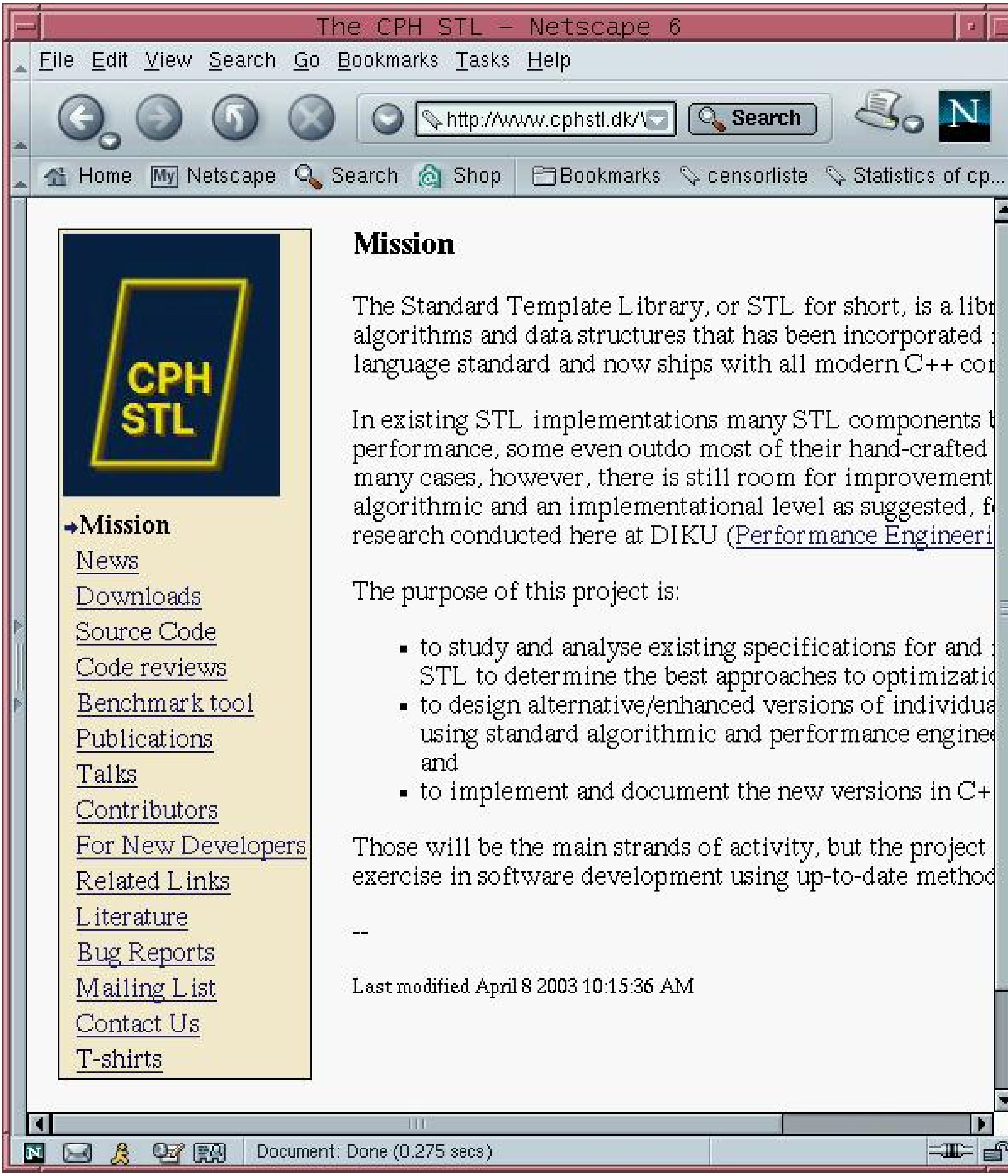
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The Standard Template Library, or STL for short, is a libr
algorithms and data structures that has been incorporated i
language standard and now ships with all modern C++ co
In existing STL implementations many STL components t
performance, some even outdo most of their hand-crafted
many cases, however, there is still room for improvement
algorithmic and an implementational level as suggested, f
research conducted here at DIKU (Performance Engineeri
The purpose of this project is:
to study and analyse existing specifications for and i
STL to determine the best approaches to optimizatio
to design alternative/enhanced versions of individua
using standard algorithmic and performance enginee
and
to implement and document the new versions in C+
Those will be the main strands of activity, but the project
exercise in software development using up-to-date method
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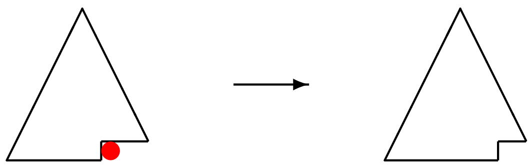
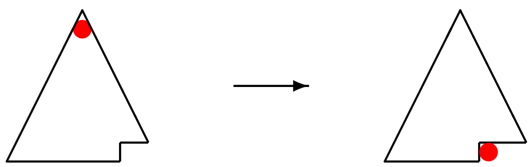
Heap functions in the STL

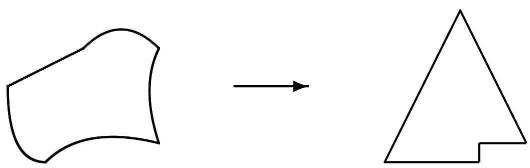
void
push_heap(position A, position Z, ordering f);

Effect:  at most $\log_2 n$ comparisons

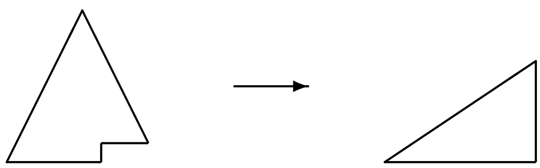
void
pop_heap(position A, position Z, ordering f);

Effect:  at most $2\log_2 n$ comparisons

void
make_heap(position A, position Z, ordering f);

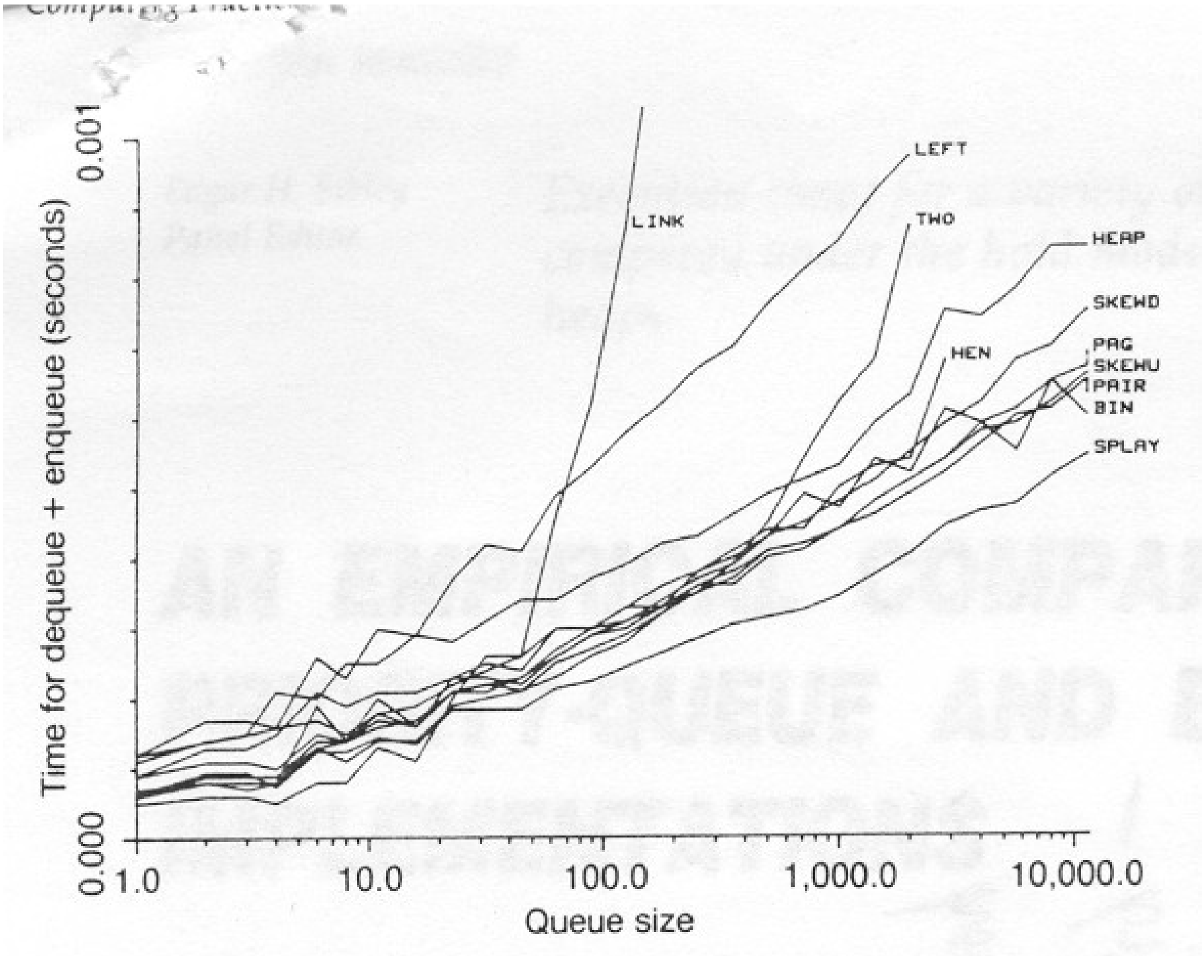
Effect:  at most $3n$ comparisons

void
sort_heap(position A, position Z, ordering f);

Effect:  at most $n\log_2 n$ comparisons

How would you do it?

Jones 1986



Operation sequence (hold model):

$push()^N[pop()push()]^K$
$e \leftarrow pop()$
increase the priority of e by $-\ln(\texttt{drand}())$
$push(e)$

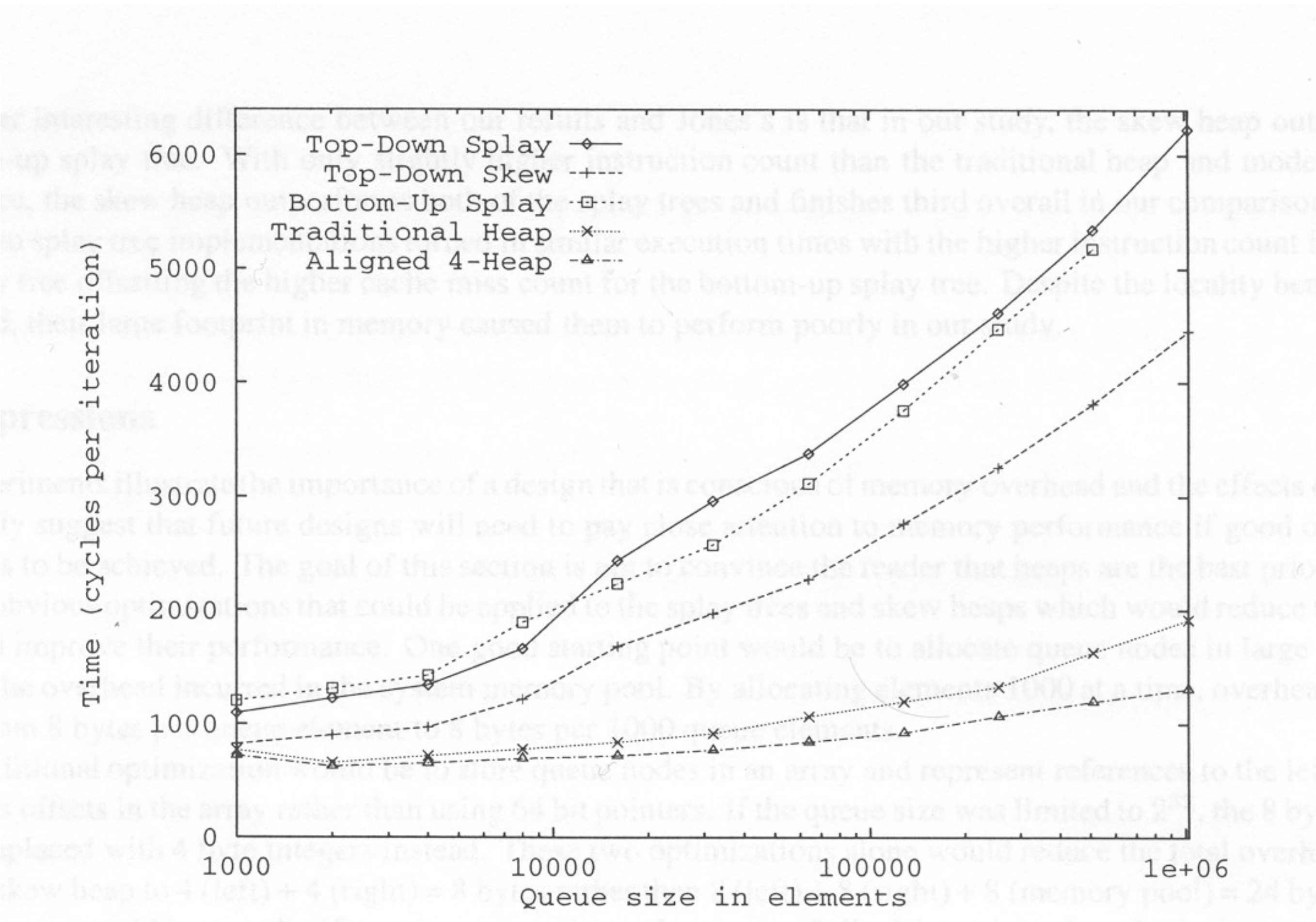
Input data:

element size: 4 B; #elements: 1–$2^{13.5}$

Environment:

computer: VAX 11/780 running UNIX (BSD 4.2); cache: 8 kB: TLB: 64 entries; compiler: Berkeley Pascal with optimization enabled

LaMarca & Ladner 1996



Operation sequence:

Hold model?

`#define NOTSORANDNUM(x) (x + RANDNUM())`

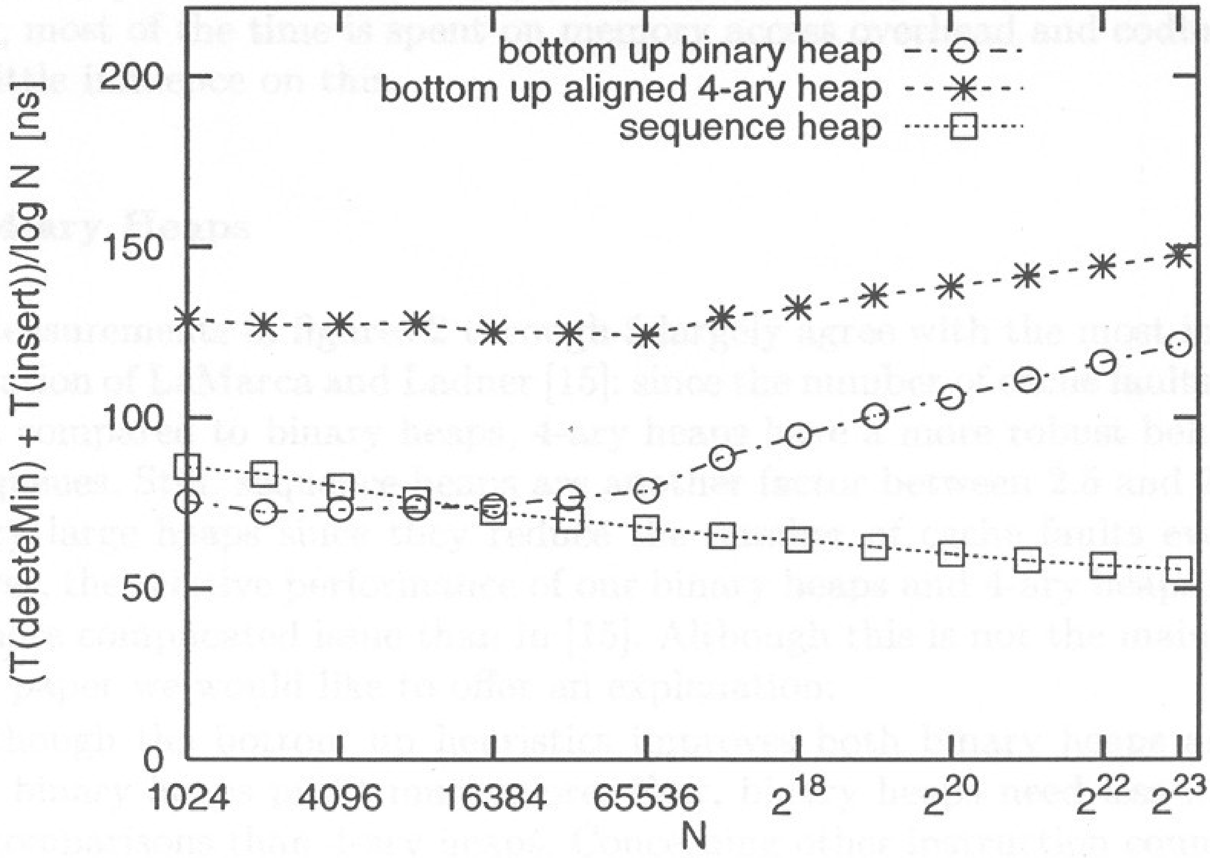
Input data:

element size: 8 B; #elements: 2^{10}–2^{23}

Environment:

computer: DEC Alphastation 250; processor: Alpha 21064A 266 MHz; L1 cache: 8 kB; L2 cache: direct-mapped, 2 MB, 32 B per line; compiler?: cc

Sanders 1999



Operation sequence:

$[push()pop()push()]^N [pop()push()pop()]^N$

Input data:

element size: 4 B, drawn randomly; satellite data: 4 B; #elements: 2^8–2^{23}

Environment:

computer: Pentium II 300 MHz; compiler `g++ -O6`

Brengel et al. 1999

Insert/Delete_min time performance of the external queues (in secs)

N [*10^6]	radix heap	array heap	buffer tree	B-tree
1	6/24	18/11	56/34	11287/259
5	17/97	74/63	148/309	66210/1389
10	35/178	353/89	201/882	-
25	85/372	724/295	311/2833	-
50	164/853	1437/645	445/6085	-
75	246/1416	2157/1005	569/9880	-
100	325/1957	2888/1408	*	-
150	478/3084	4277/2297	*	-
200	628/4036	5653/3234	*	-

Insert/Delete_min time performance of the internal queues (in secs)

N [*10^6]	Fibonacci heap	k-ary heap	pairing heap	radix heap
1	3/32	4/33	3/19	3/11
2	6/73	8/75	6/45	5/27
5	17/208	21/210	14/126	11/71
7.5	172800*/-	32/344	22/207	18/124
10	-/-	43/482	30/291	23/162
20	-/-	172800*/-	172800*/-	172800*/-

Random/Total I/Os for external queues

N [*10^6]	radix heap	array heap	buffer tree	
1	44/420	24/720	228/668	
5	422/3550	120/4560	16722/21970	
10	1124/8620	168/9440	35993/47297	
25	2780/21820	570/29520	93789/123285	
50	7798/56830	1288/66160	190147/249955	
75	12466/89370	2016/102480	286513/376625	
100	17736/124740	2776/139760	*	
150	27604/192500	4216/210080	*	
200	38284/211570	5712/284320	*	

Operation sequence:

$push()^N / pop()^N$

Input data:

element size: 4 B, drawn randomly from $[0 \,.\,.\, 10^7]$;
#elements: $1 \cdot 10^6$–$200 \cdot 10^6$

Environment:

computer: Sparc Ultra 1/143; main memory: 256 MB, 8 kB per page; local disk: 9 GB fastwide SCSI; logical block size: 64 kB; buffer size: 16 MB

Edelkamp & Stiegeler 2002

	f^0	f^1	f^2	f^3	f^4	f^5	f^6	f^7
QUICKSORT	3.86	14.59	26.73	39.04	51.47	63.44	75.68	87.89
CLEVER-QUICKSORT	3.56	12.84	23.67	33.16	43.80	54.37	64.62	75.08
BOTTOM-UP-HEAPSORT	5.73	13.49	22.60	32.05	41.59	51.14	60.62	70.11
MDR-HEAPSORT	7.14	15.39	24.37	33.82	43.04	52.63	61.87	71.02
WEAK-HEAPSORT	7.15	14.89	23.66	32.82	41.97	51.08	60.13	69.27
RELAXED-WEAK-HEAPSORT	8.29	15.96	24.63	33.51	42.32	51.33	60.06	68.83
GREEDY-WEAK-HEAPSORT	9.09	16.60	25.24	34.12	43.03	51.77	60.72	69.78
QUICK-HEAPSORT	6.35	15.89	26.20	36.98	47.59	58.16	69.37	79.59
QUICK-WEAK-HEAPSORT	6.06	14.49	23.86	33.49	43.30	52.99	62.85	72.54
CLEVER-HEAPSORT	5.30	14.01	23.66	33.65	43.95	53.79	63.60	73.94
CLEVER-WEAK-HEAPSORT	5.97	13.82	22.83	31.95	41.31	50.40	59.58	69.61

Operation sequence:

$make(N)[pop()]^N$

Input data:

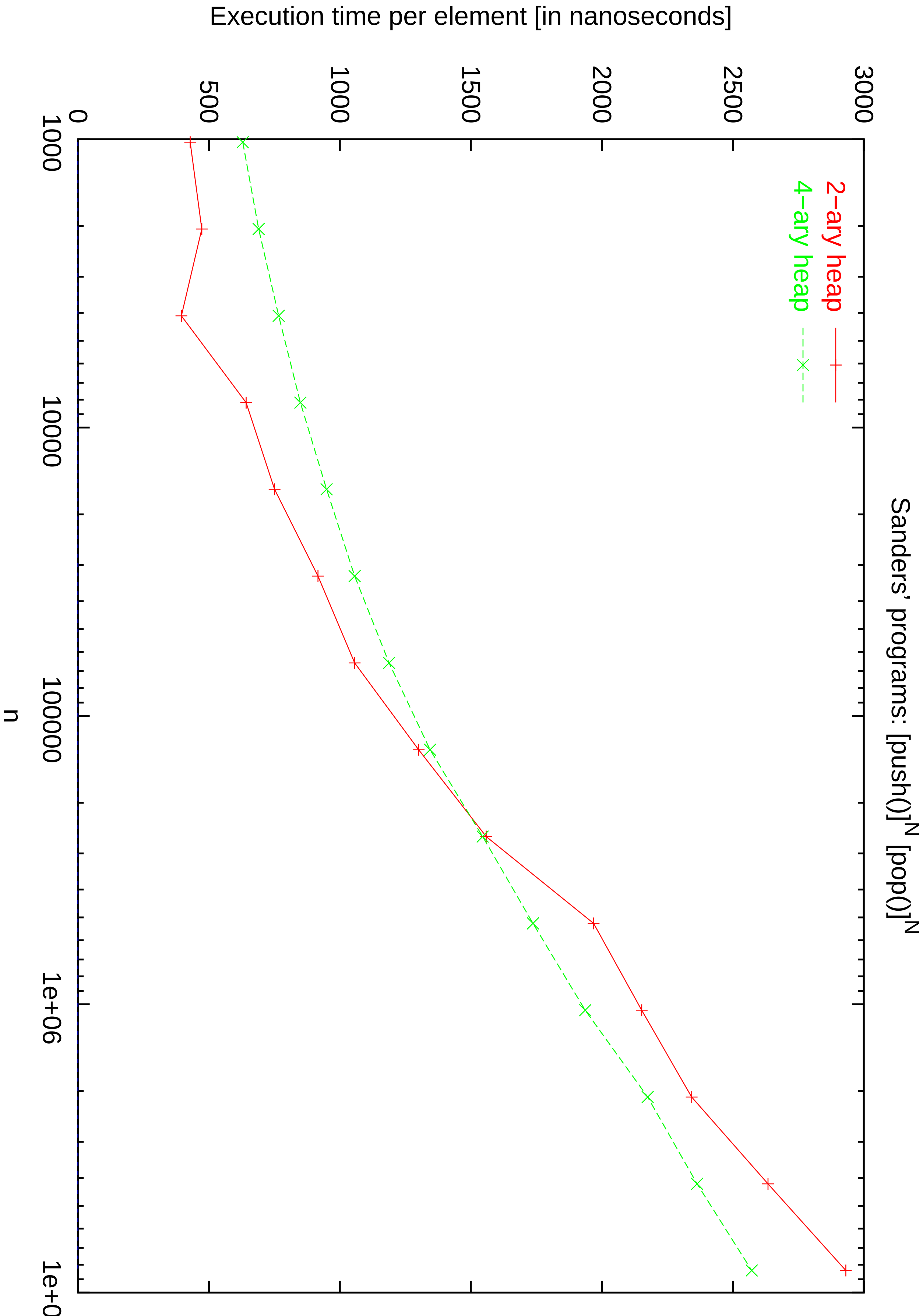
element size: 4 B, floating point numbers drawn randomly; #elements: 10^6; ordering: $f^0(x) = x$ and $f^i(x) = \ln(f^{i-1}(x+1))$ for $i > 0$

Environment:

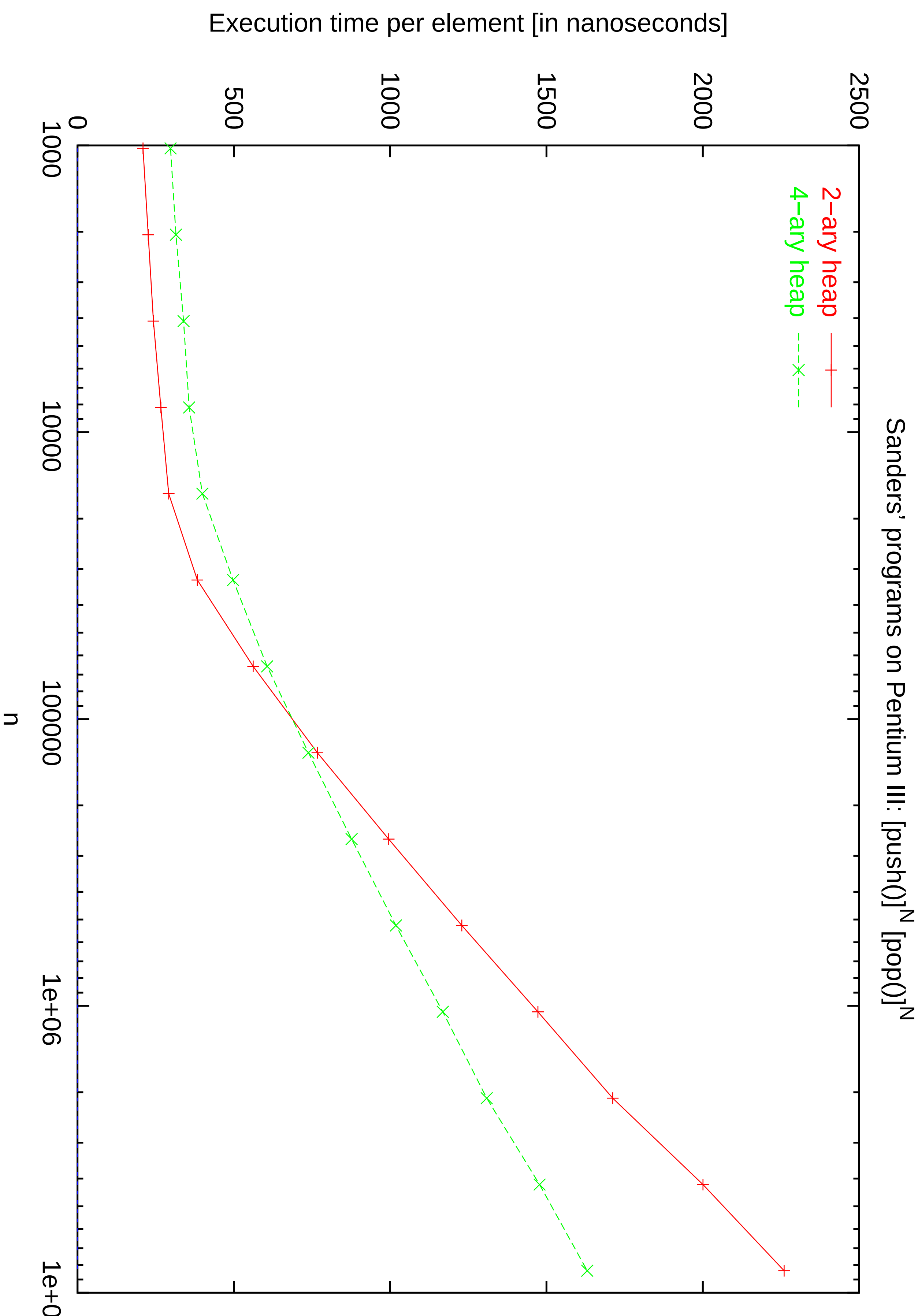
computer: Pentium III 450 MHz; compiler `g++ -O2`

How would you do it now?

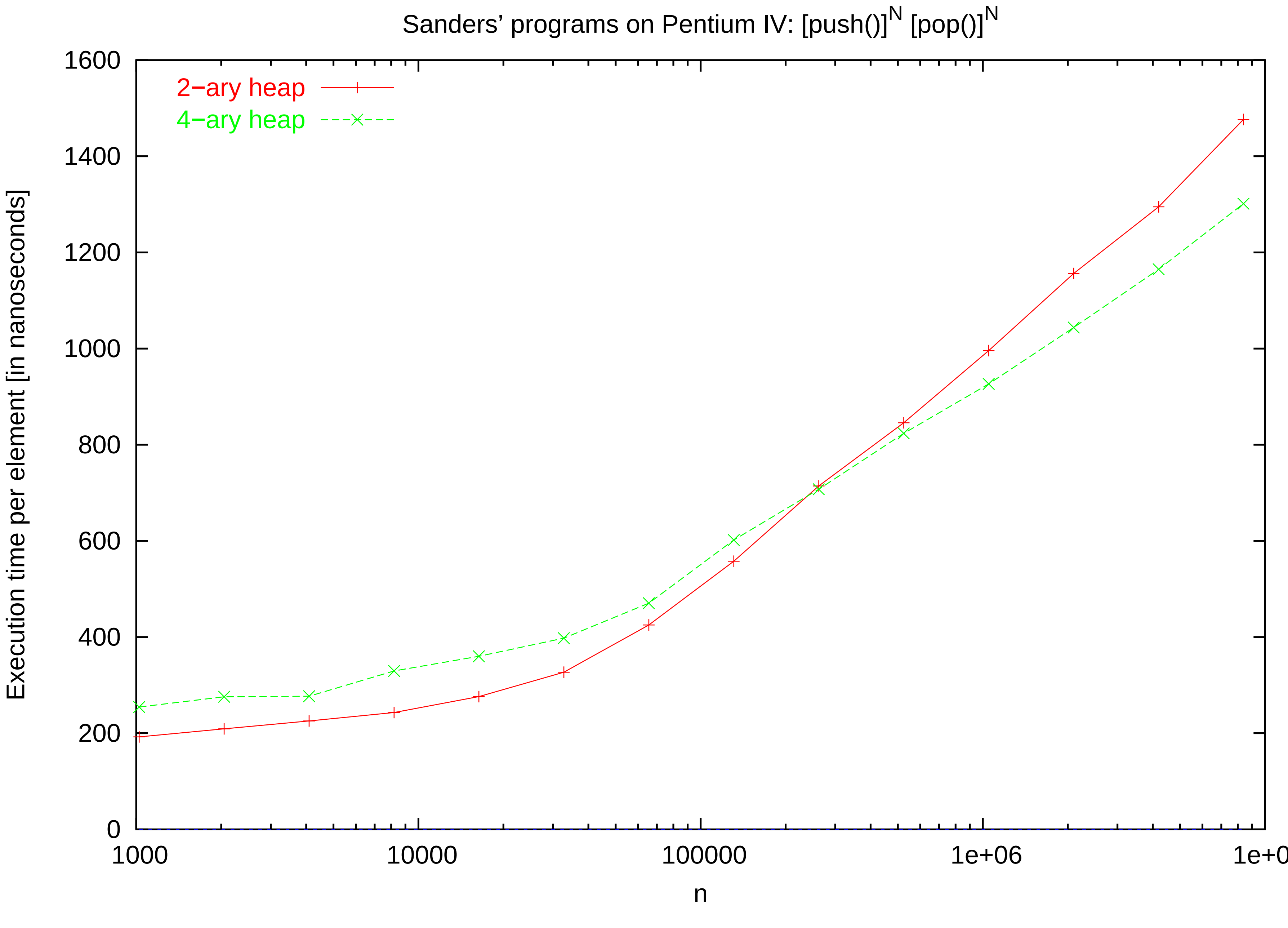
Sanders' programs on Pentium II



Sanders' programs on Pentium III



Sanders' programs on Pentium IV



Cost of unsigned int operations

initializations	instruction	unsigned int
$p \leftarrow 1$ $a[i] \leftarrow 0$ $x \leftarrow 2^{20}$	$a[i] \leftarrow x$	$n = 2^{10} .. 2^{24}$ 4.1–4.7 ns
$p \leftarrow 617$ $a[i] \leftarrow 0$ $x \leftarrow 2^{20}$	$a[i] \leftarrow x$	$n = 2^{10} .. 2^{14}$ 7.3–8.9 ns $n = 2^{15}$ 12 ns $n = 2^{16}$ 29 ns $n = 2^{16} .. 2^{22}$ 62–63 ns
$p \leftarrow 1$ $a[i] \leftarrow 0$ $x \leftarrow 2^{20}$	$x \leftarrow a[i]$	$n = 2^{10} .. 2^{24}$ 3.3–3.8 ns
$p \leftarrow 617$ $a[i] \leftarrow 0$ $x \leftarrow 2^{20}$	$x \leftarrow a[i]$	$n = 2^{10} .. 2^{15}$ 3.3–4.1 ns $n = 2^{16}$ 23 ns $n = 2^{17} .. 2^{22}$ 45–55 ns
$p \leftarrow 1$ $a[i] \leftarrow 0$ $x \leftarrow 2^{20}$	$r \leftarrow (a[i] < x)$	$n = 2^{10} .. 2^{24}$ 5.3–5.8 ns
$p \leftarrow 1$ $a[i] \leftarrow 0$ $x \leftarrow 2^{20}$	$r \leftarrow (\ln(a[i]) < \ln(x))$	$n = 2^{10} .. 2^{24}$ 580–610ns

Cost of bigint operations

initializations	instruction	bigint
$p \leftarrow 1$ $a[i] \leftarrow 0$ $x \leftarrow 2^{20}$	$a[i] \leftarrow x$	$n = 2^{10} \,..\, 2^{21}$ 60–66 ns $n = 2^{22}$ 290 ns
$p \leftarrow 617$ $a[i] \leftarrow 0$ $x \leftarrow 2^{20}$	$a[i] \leftarrow x$	$n = 2^{10} \,..\, 2^{12}$ 75–78 ns $n = 2^{13}$ 117 ns $n = 2^{14}$ 229 ns $n = 2^{15} \,..\, 2^{20}$ 297–318 ns $n = 2^{21} \,..\, 2^{22}$ 748–752 ns
$p \leftarrow 1$ $a[i] \leftarrow 0$ $x \leftarrow 2^{20}$	$x \leftarrow a[i]$	$n = 2^{10} \,..\, 2^{22}$ 18–21 ns
$p \leftarrow 617$ $a[i] \leftarrow 0$ $x \leftarrow 2^{20}$	$x \leftarrow a[i]$	$n = 2^{10} \,..\, 2^{12}$ 24 ns $n = 2^{13}$ 83 ns $n = 2^{14}$ 180 ns $n = 2^{15} \,..\, 2^{22}$ 230–260 ns
$p \leftarrow 1$ $a[i] \leftarrow 0$ $x \leftarrow 2^{20}$	$r \leftarrow (a[i] < x)$	$n = 2^{10} \,..\, 2^{22}$ 13–16 ns

Other current research

Pointer-based methods:

hopelessly slow

$\longrightarrow$ theoretical computer science

Methods with good amortized bounds:

terrible worst case

$\longrightarrow$ not relevant for us

Methods with few element moves:

bad cache behaviour

$\longrightarrow$ not good for us

External-memory methods:

high constants

$\longrightarrow$ relevant only for very large data sets

Cache-oblivious methods:

huge constants

$\longrightarrow$ theoretical computer science

Our policy-based framework

```
template <arity d, typename position, typename ordering>
class heap_policy {
public:
  typedef typename
    std::iterator_traits<position>::difference_type index;
  typedef typename
    std::iterator_traits<position>::difference_type level;
  typedef typename
    std::iterator_traits<position>::value_type element;

  template <typename integer>
  heap_policy(integer n = 0);

  bool is_root(index) const;
  bool is_first_child(index) const;
  index size() const;
  level depth(index) const;
  index root() const;
  index leftmost_leaf() const;
  index last_leaf() const;
  index first_child(index) const;
  index parent(index) const;
  index ancestor(index, level) const;
  index top_some_absent(position, index,
    const ordering&) const;
  index top_all_present(position, index,
    const ordering&) const;
  void update(position, index, const element&);
  void erase_last_leaf(position, const ordering&);
  void insert_new_leaf(position, const ordering&);
private:
  index n;
};
```

Input data

	cheap move	expensive move
cheap comparison	unsigned int	bigint
expensive comparison	unsigned int In comparison	(int, bigint) In comparison

One new old idea: local heaps

Our solution for sort_heap()

In-place mergesort by Katajainen, Pasanen, and Teuhola [1996]

Fine-tuning not yet implemented

Almost as fast as quicksort, see CPH STL Report 2003-2

Our solution for make_heap()

Depth-first heap construction by Bojesen, Katajainen, and Spork [2000]

Almost optimal in all respects

Other work:

less element comparisons

$\longrightarrow$ theoretical computer science

Various approaches for pop_heap()

– top-down → many element comparisons

– bottom-up → typical case good

– move-saving bottom-up → theoretical computer science

– binary-search top-down

– two-levels-at-a-time top-down

Various approaches for push_heap()

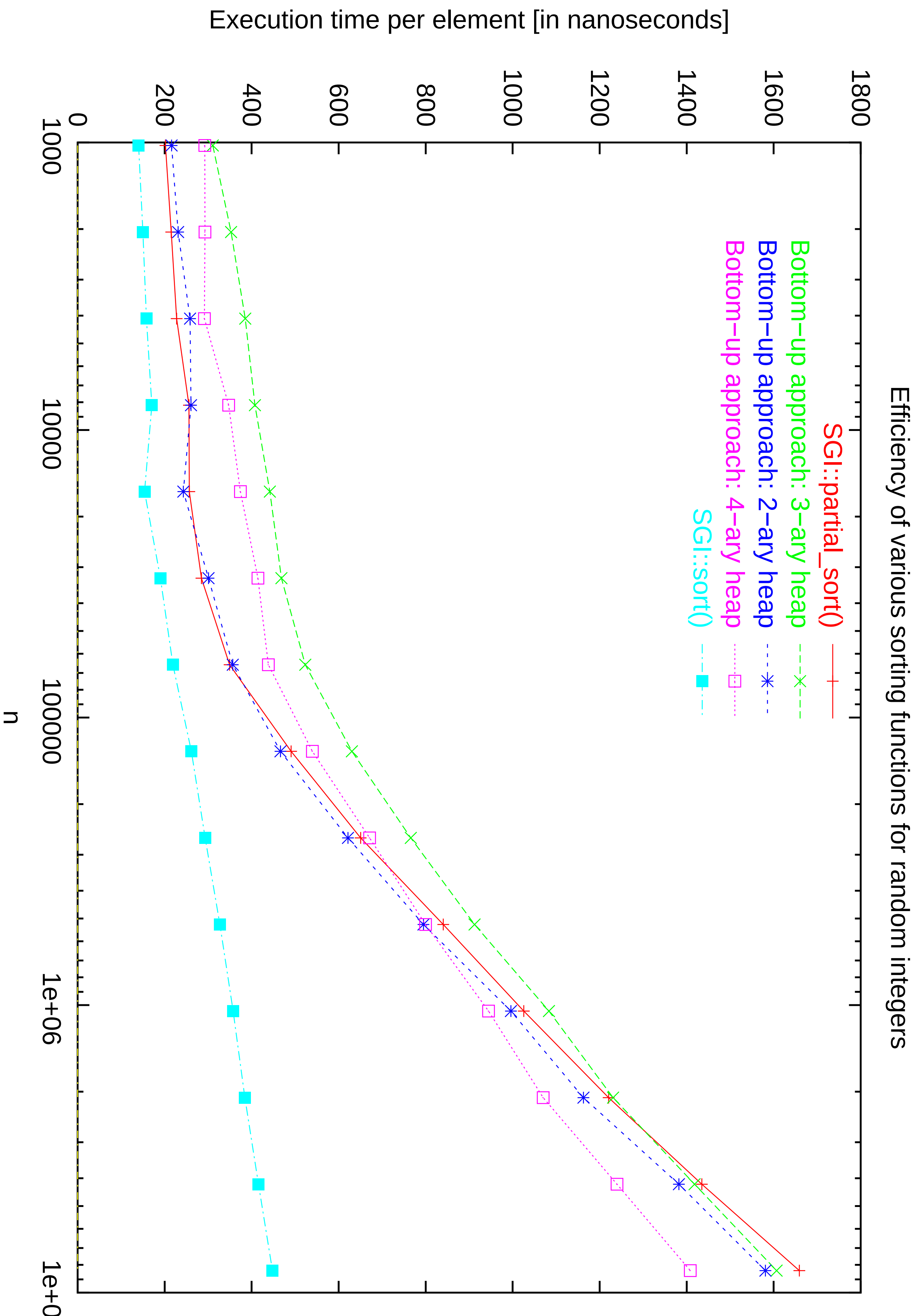
– move-saving top-down → slow

– bottom-up → typical case good

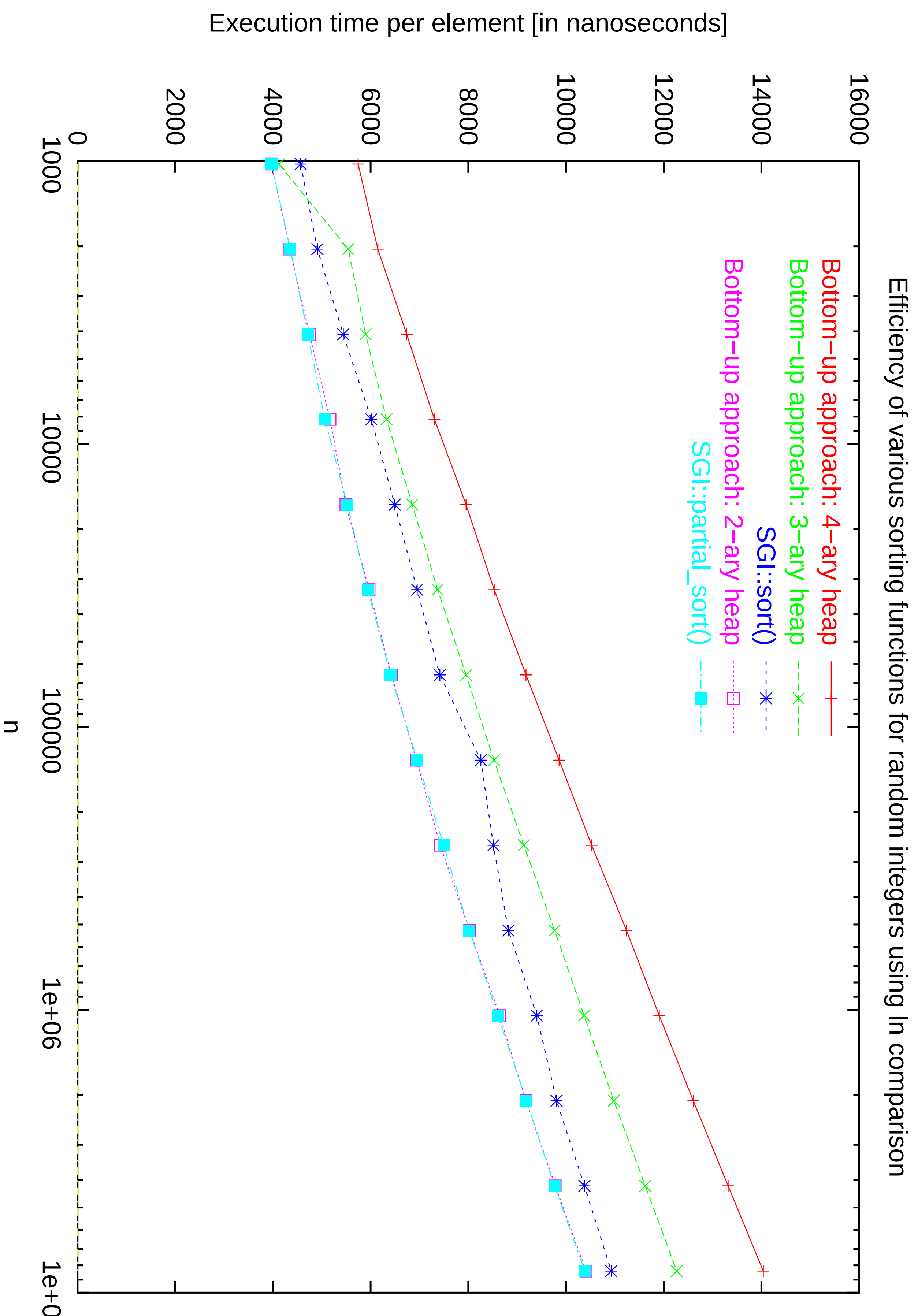
– bottom-up with buffering → complicated

– binary-search bottom-up

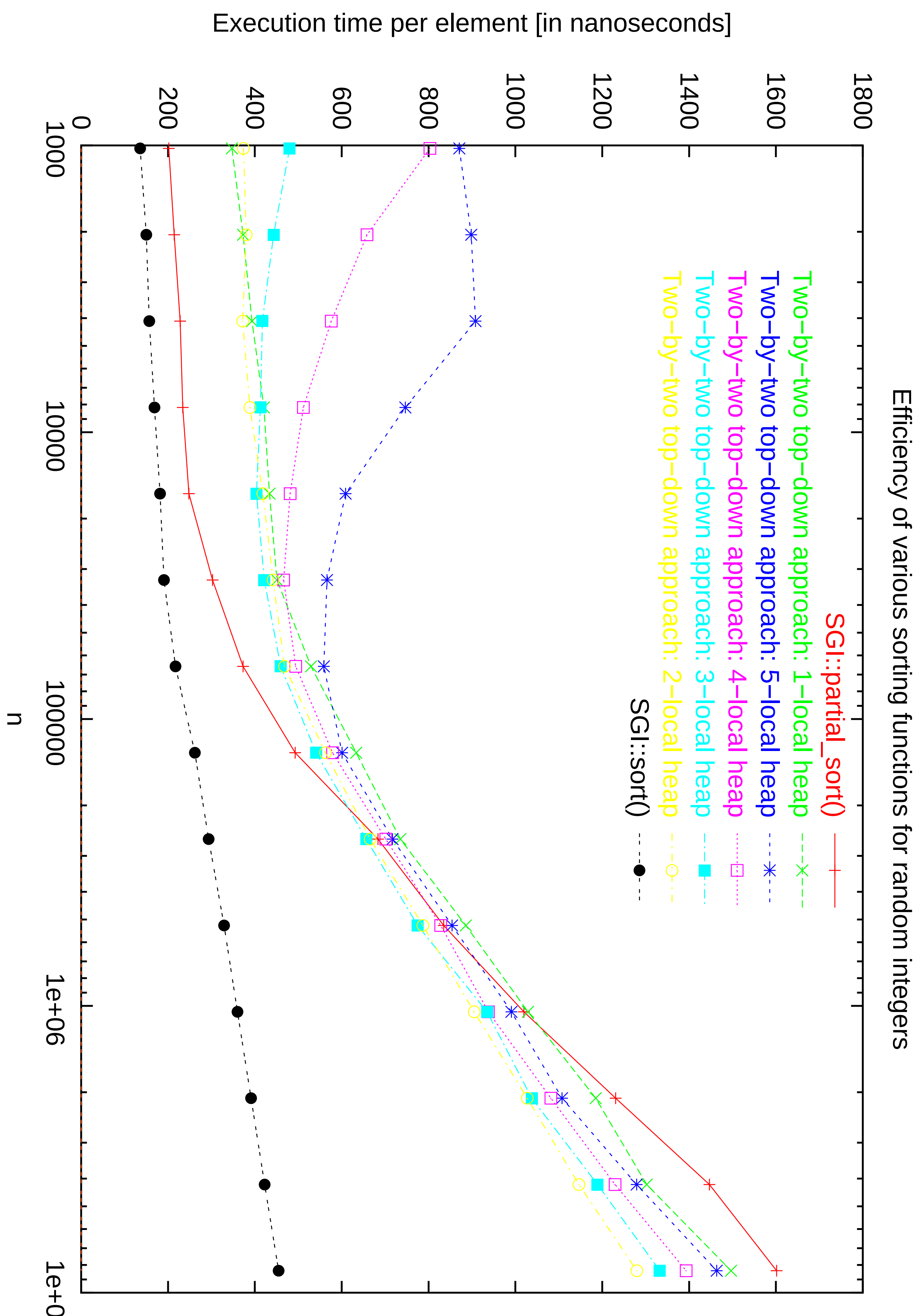
Efficiency of 2-, 3-, 4-ary heaps



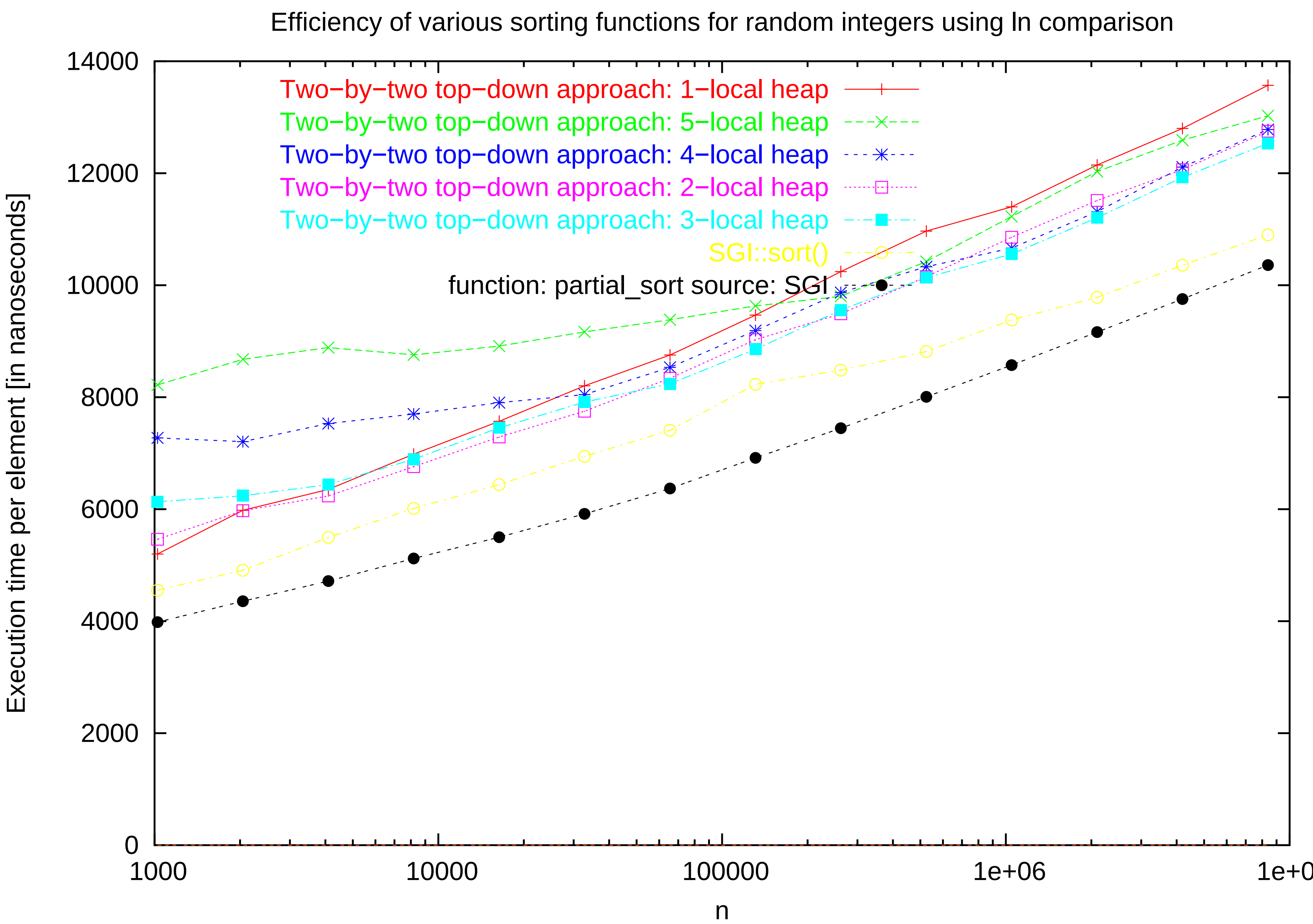
Efficiency of 2-, 3-, 4-ary heaps



Efficiency of local heaps



Efficiency of local heaps



Conclusions

– In 40 years — not much progress
– At the moment it is not clear how big the overhead of local heaps is for small problem sizes.
– Some combinations of various approaches have still to be tested.
– Code-tuning of the best approaches is still to be done.
– It takes time to develop fast library routines.
– How does technology influence on the efficiency of the library routines?

Exercise of the week

How many element comparisons incur the operation sequence

$$[push() \mid pop()]^N$$

in the worst case? Or what is the amortized complexity of each of these operations?

$1.5N \log_2 N$ is an obvious upper bound and $N \log_2 N$ an obvious lower bound.

Recall that the operation sequence

$$make(N)[pop()]^N$$

requires about $1.5N \log_2 N$ element comparisons.